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# Penetration of Electromagnetic Pulses through Larger Apertures in Shielded Enclosures

Dikewood Industries, Inc.

May 1976

**AFWL-TR-75-95** 

AFWL-TR-75-95



# PENETRATION OF ELECTROMAGNETIC PULSES THROUGH LARGER APERTURES IN SHIELDED ENCLOSURES

University of !llinois Urbana, IL 61801

for Dikewood Corporation Albuquerque, NM 87106

May 1976

**Final Report** 

Approved for public release; distribution unlimited.

This research was sponsored by the Defense Nuclear Agency under Subtask EB088, Work Unit 33, Work Unit Title: Coupling Characteristics of Apertures.

Prepared for Director DEFENSE NUCLEAR AGENCY Washington, DC 20305

AIR FORCE WEAPONS LABORATORY Air Force Systems Command Kirtland Air Force Base, NM 87117

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#### AFWL-TR-75-95

This final report was prepared by the University of Illinois, Urbana, IL, for Dikewood Corporation, Albuquerque, NM, under Contract F29601-74-C-0010, Job Order WDNE 2705, with the Air Force Weapons Laboratory, Kirtland AFB, NM. Mr. William D. Prather (ELP) was the Laboratory Project Officer-in-Charge.

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SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION	READ INSTRUCTIONS BEFORE COMPLETING FORM			
L REPORT HUMBER AFWL-TR-75-95	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER		
4. TITLE (and Subtitie)		5. TYPE OF REPORT & PERIOD COVERED		
PENETRATION OF ELECTROMAGNETIC PULS	Final Report			
LARGER APERTURES IN SHIELDED ENCLOS	6. PERFORMING ORG. REPORT NUMBER			
7. Author(s) R. Mittra		8. CONTRACT OR GRANT NUMBER(a)		
L. Wilson Pearson University of Illinois	F29601-74-C-0010			
PERFORMING ORGANIZATION NAME AND ADDRESS		10 PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS		
Dikewood Corporation Albuquerque, New Mexico 87106	62707H; WDNE2705			
1. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE		
Director	May 1976			
Defense Nuclear Agency Washington, DC 20305	13. NUMBER OF PAGES			
14. MONITORING AGENCY NAME & ADDRESS(If different	from Controlling Office)	15. SECURITY CLASS (of this report)		
Air Force Weapons Laboratory (ELP)	UNCLASSIFIED			
Kirtland AFB, New Mexico 87117	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE			

16. DISTRIBUTION STATEMENT (of this Report

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This research was sponsored by the Defense Nuclear Agency under Subtask EB088, Work Unit 33, Work Unit Title: Coupling Characteristics of Apertures.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electromagnetic Fields and Waves Interaction and Coupling Aperture Penetration

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The results of an initial investigation of the Singularity Expansion Method representation of the electromagnetic coupling through a rectangular aperture in a perfectly conducting sheet are reported. The problem is formulated in terms of the coupled Hallen-type integral equations for the dual problem of a rectangular plate. The integral equations are converted to a system of linear algebraic equations by way of the method of moments with subsectionally constant expansion functions and collocation testing. Several techniques used in

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#### 20. ABSTRACT (Continued)

minimizing the execution time of the computations are described. Some difficulties in accurately approximating the singularities of the system of integral equations by the singularities of the algebraic system are discussed. These difficulties arise because the subsectionally constant representation for the current cannot adequately represent the correct edge singularities in the currents on the plate. A set of pole trajectories indicative of the trends in pole location for the plate is reported. A listing of the pertinent computer code is provided.

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#### SECTION I

#### INTRODUCTION

This report presents the results of an investigation for representing the transient electromagnetic coupling through a rectangular aperture by means of the singularity expansion method.

The singularity expansion method, which was introduced by Baum in 1971 (ref. 1), has been subsequently applied to many scatterer geometries. The essence of the singularity expansion method is the representing of the temporal response of a body in terms of the complex natural frequencies for the body.

Taylor et al. point out that the singularity expansion for an aperture in an infinite perfectly conducting screen can be determined in terms of that for the complementary perfectly conducting plate by way of Babinet's principle (ref. 2). This approach was taken in the work reported here. The remaining discussion is directed to the equivalent problem of determining the current distributions on the complementary plate geometry.

Rahmat-Samii and Mittra have derived a coupled pair of Hallen-type integral equations governing the current behavior on the rectangular plate (ref. 3). The work reported here builds on their work by generalizing the integral equations and solution method to the complex frequency plane for the

<sup>1.</sup> Baum, C. E., "On the Singularity Expansion Method for the Solution of Electromagnetic Interaction Problems," Interaction Note 88, Air Force Weapons Laboratory, Kirtland AFB, NM, December 1971.

Taylor, C. D., Crow, T. T., and Chen, K-T, "On the Singularity Expansion Method Applied to Aperture Penetration: Part I Theory," Interaction Note 134, Air Force Weapons Laboratory, Kirtland AFB, NM, May 1973.

<sup>3.</sup> Rahmat-Samii, Y. and Mittra, R., "Integral Equation Solution and RCS Computation of a Thin Rectangular Plate," Interaction Note 156, Air Force Weapons Laboratory, Kirtland AFB, NM, December 1973.

SEM application. The same method-of-moments formulation, as described in (ref. 3), is used, i.e., two-dimensional pulse expansion functions with collocation testing.

In order that the computation time be practical in a problem of this complexity, a great deal of care was given to algorithmic streamlining in the conduct of this work. The streamlining includes maximum exploitation of geometric symmetry, organization of calculations to make use of redundant terms and partial terms occurring in the calculation, and direct algorithmic exploitation of matrix sparseness. The end result is a highly efficient computer code. Key features of the algorithms are discussed in this report.

The pulse expansion appears to be inadequate in accurately modeling the rectangular plate. The difficulty, which relates to representing the actual size of the plate, is demonstrated and discussed herein. Remedies for the problem are suggested, but they are outside the scope of the present work.

By holding the zoning scheme invariant while the aspect ratio of the plate was changed, self-consistent pole trajectories for the four fundamental modes were determined. For the reasons cited above, these poles cannot claim to be the exact poles for the body. They are, however, indicative of the trends in the pole behavior for the plate under change in aspect ratio. These results are reported and discussed in this context.

#### SECTION II

# THIN-PLATE INTEGRAL EQUATION FORMULATION FOR COMPLEX WAVENUMBER

Rahmat-Samii and Mittra (ref. 3) give an integral equation formulation for the rectangular plate subject to time-harmonic excitation. Their results may be directly extended to the complex wavenumber case. That is, for the geometry in Figure 1 with exp[st] time dependence,  $s = \sigma + j\omega$  complex, and an incident plane-wave magnetic field component

 $\overline{H}^{i} = [H_{ox}^{i} \hat{u}_{x} + H_{oy}^{i} \hat{u}_{y} + H_{oz}^{i} \hat{u}_{z}] \exp[j(k_{x} x + k_{y} y + k_{z} z)]$  the following coupled integral equations result:

$$\int_{-L/2}^{L/2} \int_{-W/2}^{W/2} \left( \int_{y}^{J_{x}(x,y)} \int_{y}^{J_{x}(x,y)} K(x,y|x',y') dx' dy' = \frac{i}{k_{z}} \left( \int_{-Ho_{x}^{i}}^{Ho_{x}^{i}} dy' \right) \exp[j(k_{x} + k_{y} + k_{y})]$$

$$+ \frac{\pi}{k} \begin{cases} -1 \\ -j \end{cases} \sum_{-\infty}^{\infty} C_{n} [j^{n+1} \exp[j(n+1)\phi] J_{n+1}(-s\rho/c) \\ + \begin{cases} 1 \\ -1 \end{cases} j^{n-1} \exp[j(n-1)\phi] J_{n-1}(-s\rho/c)]$$
 (1)

The kernel is given by

$$K(x,y \mid x',y') = \exp[-sR/c]/R$$
 (2)

with

$$R = [(x - x^{\dagger})^{2} + (y - y^{\dagger})^{2}]^{1/2}$$

The  $J_{x}(x,y)$  and  $J_{y}(x,y)$  denote the respective x and y components of current on the plate;  $J_{n}(\zeta)$  denotes the Bessel function of the first kind; the  $C_{n}$  are unknown constants; c is the velocity of light; and  $(\rho,\phi)$  are the polar coordinates for the point (x,y) on the plate. Equation (1) holds for  $x \in (-L/2,L/2)$  and  $y \in (-w/2,w/2)$ , and z = 0.

It is pointed out that the two integral equations represented by (1) are

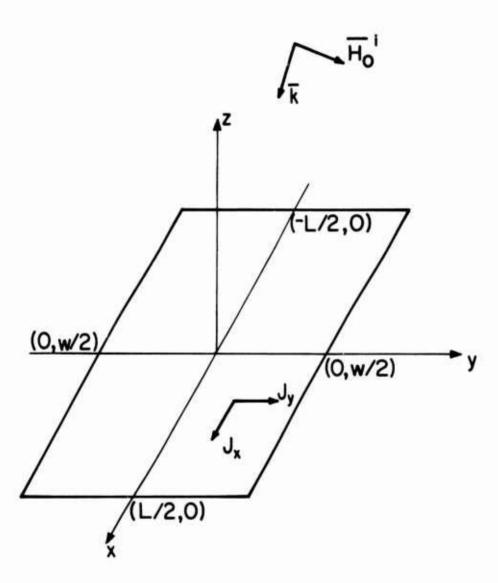


Figure 1. Geometry of the Rectangular Plate

mation is simply a Bessel function expansion of the homogeneous solution to the wave equation which occurs in the derivation of (1). Details of arriving at this expansion are found in (ref. 3). The pair of integral equations (1) is complete in the sense that no additional constraints are needed to correctly specify the currents. It is noteworthy, however, that current solutions to (1) satisfy the Meixner's edge condition (ref. 4). Namely,

$$J_{\mathbf{x}}[\pm(L/2 - d), y] \rightarrow d^{1/2}$$

$$J_{\mathbf{y}}[\pm(L/2 - d), y] \rightarrow d^{-1/2}$$

$$J_{\mathbf{x}}[\mathbf{x}, \pm(w/2 - d)] \rightarrow d^{-1/2}$$

$$J_{\mathbf{y}}[\mathbf{x}, \pm(w/2 - d)] \rightarrow d^{1/2}$$

$$J_{\mathbf{y}}[\mathbf{x}, \pm(w/2 - d)] \rightarrow d^{1/2}$$
(3)

The ability of a numerical solution to approximate the behavior of eqn. (3) is a key point in a subsequent discussion.

<sup>4.</sup> Bladel, J. Van, Electromagnetic Fields, McGraw-Hill, New York, pp. 385-387, 1964.

#### SECTION III

#### SYMMETRY CONDITIONS FOR THE NATURAL MODE CURRENTS

The natural frequencies of (1) occur when the complex frequency s is such that there are non-trivial  $J_x$  and  $J_y$  and the accompanying  $C_n$  satisfying (1) for  $\overline{H}^1$  = 0. Such  $J_x$  and  $J_y$  solutions are natural mode current solutions for the rectangular plate, and the concomitant value of s is a pole of the plate. The vanishing of incident wave dependence gives rise to symmetry in the integral equations. By discerning the symmetry relations a priori and bringing them to bear upon solution procedures, one gains significant computational savings in the numerical solution for poles and natural modes. These symmetry relations are explored in this section.

The excitation-free form of (1) is

$$\int_{-L/2}^{L/2} \int_{x}^{w/2} J_{x} K(x,y|x',y') dx' dy' = \frac{j\pi c}{s} \sum_{-\infty}^{\infty} C_{n} \left( j^{n+1} \exp[j(n+1)\phi] J_{n+1} (-s\rho/c) + j^{n-1} \exp[j(n-1)\phi] J_{n-1} (-s\rho/c) \right)$$
(4a)

and

$$\int_{-L/2}^{L/2} \int_{y}^{w/2} J_{y} K(x,y|x',y') dx' dy' = \frac{\pi c}{s} \int_{-\infty}^{\infty} C_{n} \left\{ j^{n+1} \exp[j(n+1)\phi] J_{n+1} (-s\rho/c) - j^{n-1} \exp[j(n-1)\phi] J_{n-1} (-s\rho/c) \right\}$$
(4b)

By using the symmetry of the Bessel function with respect to order, expanding the exponentials by way of Euler's identity, and appropriately adjusting the indices, one arrives at the following equation after some manipulation.

$$\begin{split} & \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} \\ & = \frac{j\pi c}{s} \sum_{n=0}^{\infty} \left( j^{n+1} d_n^+ \left[ \cos(n+1)\phi J_{n-1}(-s\rho/c) - u_{n-1} \cos(n-1)\phi J_{n-1}(-s\rho/c) \right] \right. \\ & \left. - j^n d_n^- \left[ \sin(n+1)\phi J_{n+1}(-s\rho/c) - \sin(n-1)\phi J_{n-1}(-s\rho/c) \right] \right\} \\ & \text{and} \\ & \frac{L/2}{s} \int_{-L/2}^{W/2} \int_{-W/2}^{W/2} J_y K dx' dy' \\ & = \frac{j\pi c}{s} \sum_{n=0}^{\infty} \left( j^{n+1} d_n^+ \left[ \sin(n+1)\phi J_{n+1}(-s\rho/c) + \sin(n-1)\phi J_{n-1}(-s\rho/c) \right] \right. \\ & \left. + j^n d_n^- \left[ \cos(n+1)\phi J_{n+1}(-s\rho/c) + u_{n-1} \cos(n-1)\phi J_{n-1}(-s\rho/c) \right] \right) \end{split}$$

where

$$d_n^{\pm} = C_n \pm C_{-n}$$

and

$$u_n = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

It is noted that the  $d_n^+$  multiply terms containing cosine functions in the  $J_x$  equation, while they multiply terms containing sine functions in the  $J_y$  equation. The situation is reversed for the  $d_n^-$ .

Because of the symmetry properties of the kernel, the integral operator on the left-hand sides of (5) produces a function whose symmetry character is identical to that of the current on which it operates. Then, for a given current symmetry, only part of the  $d_n^{\pm}$  on the right-hand side may be nonzero because of the symmetries possessed by the trigonometric terms. Thus, the respective symmetries for  $J_x$  and  $J_y$ , which are compatible, and the

surviving terms in the right-side series may be discerned by 1) postulating a symmetry for  $J_x$ , 2) determining from (5a) which right-hand side terms survive so as to be compatible with the  $J_x$  symmetry, 3) observing in (5b) the variation which terms have non-zero coefficients, and 4) determining the  $J_y$  symmetry conditions compatible with the postulated  $J_x$  symmetry conditions.

For example, if  $J_X$  is symmetric with respect to the y axis and antisymmetric with respect to the x axis, only  $\sin(n+1)\phi$  terms with n even are compatible in  $(5\omega)$ . Thus, only  $d_n$ , n even, may be non-zero. In the right-hand side of (5b), the coefficients multiply  $\cos(n+1)\phi$  terms with n even. These cosines sum to functions which are antisymmetric with respect to the y axis and symmetric with respect to the x axis. Stated mathematically, if

$$J_{\mathbf{y}}(\mathbf{x},\mathbf{y}) = J_{\mathbf{y}}(-\mathbf{x},\mathbf{y}) \tag{6a}$$

and

$$J_{x}(x,y) = -J_{x}(x,-y)$$
 (6b)

then

$$d_n^+ = 0$$
, for all n, (6c)

$$d_n = 0, \quad n \text{ odd}, \tag{6d}$$

and

$$J_{\mathbf{v}}(\mathbf{x},\mathbf{y}) = -J_{\mathbf{v}}(-\mathbf{x},\mathbf{y}) \tag{6e}$$

$$J_{y}(x,y) = J_{y}(x,-y)$$
 (6f)

These vector symmetries are in accord with the general symmetry relations given by Baum (ref. 5). The information in (6) may be used to reduce the complexity of the integral equations (4), viz., by (6a,b,e,f) the range of each integration may be halved while by (6c,d) the zero terms of the right-hand side are known a priori:

$$\int_{0}^{L/2} \int_{0}^{w/2} \int_{x}^{J} \int_{x}^{L+1} (x,y|x',y') dx' dy'$$

$$= \frac{\pi c}{s} \int_{n=0}^{\infty} d_{n}^{-1} \int_{0}^{n-1} \left[ \sin(n+1)\phi J_{n-1}(-s\rho/c) - \sin(n-1)\phi J_{n-1}(-s\rho/c) \right]$$
(7a)

n even

<sup>5.</sup> Baum, C. E., "Interaction of Electromagnetic Fields with any Object which has an Electromagnetic Symmetry Plane," Interaction Note 63, Air Force Weapons Laboratory, Kirtland AFB, NM, March 1971.

and

$$\int_{0}^{L/2} \int_{0}^{\omega/2} \int_{0}^{J_{y}} K^{+-}(x,y|x',y') dx' dy'$$

$$= \frac{\pi c}{s} \sum_{n=0}^{\infty} \int_{0}^{m+1} d_{x}^{-1} \left[\cos(n+1)\phi \int_{0}^{J_{n+1}} (-\omega/c) + u_{n-1} \cos(n-1)\phi \right]_{n-1}^{J_{n-1}} (-s\rho/c)$$
(7b)

The even

where

$$K^{+-}(x,y|x',y') = K(x,y|x',y') - K(x,y|-x',y') + K(x,y|x',-y') - K(x,y|-x',-y')$$
(8a)

and

$$K^{-+}(x,y|x',y') = K(x,y|x',y') + K(x,y|-x',y') - K(x,y|x',-y') - K(x,y|-x',-y')$$
(85)

For subsequent reference

$$K^{++}(x,y|x',y') = K(x,y|x',y') + K(x,y|-x',y') + K(x,y|x',-y') + K(x,y|-x',-y')$$
(8c)

and

$$K^{-}(x,y|x',y') = K(x,y|x',y') - K(x,y|-x',y') - K(x,y|x',-y') + K(x,y|-x',-y')$$
(8d)

are defined as well. Equations (7) are enforced for  $z=0, x \in (0,L/2)$  and  $y \in (0,w/2)$ .

Table 1 summarizes the four symmetry cases which are derived as in the foregoing discussion. By means of this table, four integral equation pairs can be constructed in the spirit of (7) by replacing the kernels in (7) with the appropriate kernels from the table and retaining only the non-vanishing terms in the series expansion.

Figure 2 depicts qualitatively the respective modal current distributions for the lowest frequency natural reasonance exhibiting each symmetry.

Table 1

COMPATIBLE CURRENT SYMMETRY FEATURES

		-	Sym. w.r.t.	Sym. w.r.t. y axis		anti		anti		sym	
		, <b>^</b>	Kernel Sym. w.r.t. Sym. w.r.t.	x axis	anti	1	anti	E,	3,00	Sym	
			Kernel		<u>\</u>	†	<u>-</u>	+,		‡	
			Compatible Trig. Fns.		sin 2n¢		$\phi(T + uz)$ urs	cos (2n + 1) \$		cos 2n¢	
			Coefs. ≠ 0		d <sup>+</sup> 2n+1	+,	~2n	d,,	117	<sup>d</sup> 2 <sub>n+1</sub>	
			Compatible Trig. Fns.		cos 2n¢	cos (2n + 1) \$		sin (2n + 1)¢		sir 2n¢	
	٦×		Kernel	:	t ⊭	  -  -		†	!	×	
	<b>D</b>	0	x axis y axis y axis		sym	anti		sym	•	anti	
		Svm wr	x axis		Bys	sym		anti		antı	

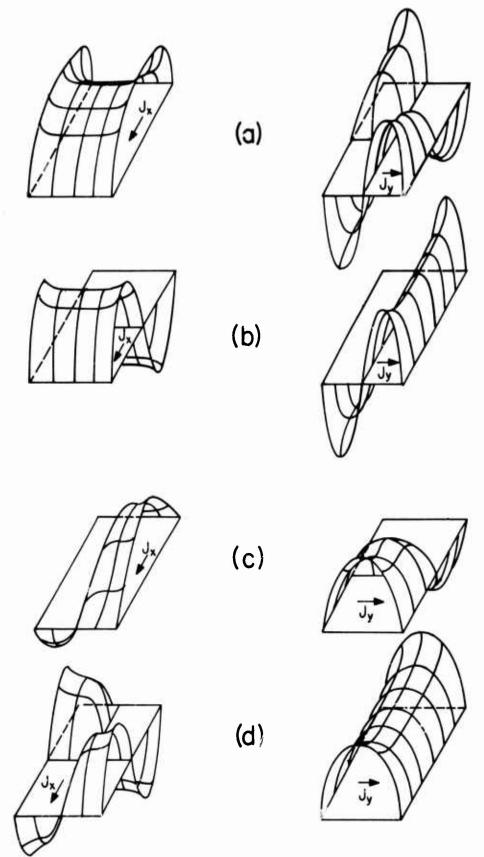


Figure 2. Lowest Order Natural Mode Current Pairs for Each of the Symmetry Cases, a) J<sub>X</sub> Symmetric w.r.t. x-Axis and Symmetric w.r.t. y-Axis, b) Symmetric-Antisymmetric, c) Antisymmetric-Symmetric, and d) Antisymmetric-Antisymmetric

#### SECTION IV

#### THE NUMERICAL MODEL

The integral equation pair of the form (7) for each of the four symmetry cases can be discretized by the method of moments. In the work reported here, two-dimensional, subsectionally constant expansion functions were used with collocation testing. The zoning scheme is represented in Figure 3.

The unknown currents  $J_x$  and  $J_y$  were expanded in piecewise constant functions as in (ref. 3) with subsectioning of the form given in Figure 3. Notice that half-width patches are used at the edges of the plate so that match points lie precisely on the edge of the plate. The half-width pulse has proved useful in realizing the actual electrical size of a body in one-dimensional problems (ref. 6). Some numerical experimentation was also done with full-sized pulses on the edges and comparative results are reported in a later section. Some difficulties occur in definition of the edge of the plate in the present formulation because of the presence of two current components which have the asymptotic behavior given in (3). This difficulty is discussed in a later section.

The boundary condition  $J_{norm}=0$  must be enforced on selected patches at the edge of the place as discussed in (ref. 3). Concomitantly, only as many  $d_n^{\dagger}$ 's are retained in the right-hand side summation in (7) as there are current values preassigned to zero. The shaded patches in Figure 3 indicate the selection of patches where a current component is preassigned a zero value. At the corner patch, both components are preassigned zero values.

<sup>6.</sup> Butler, C. M., "Integral Equation Solution Methods," in "Wire Antennas and Scatterers," Short Course Notes, University of Mississippi, April 1972. (See also IEEE Trans., v. AP-20, pp. 731-736, 1972.)

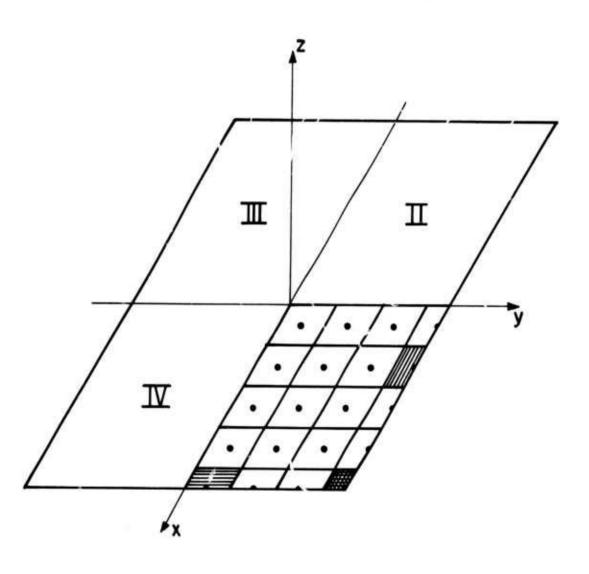


Figure 3. Subsectioning for the Discretization of the Integral Equations

By assigning one match point per expansion patch and by retaining one series expansion term for each current value preassigned in each of the two integral equations, a consistent (i.e. square) system of linear equations results. The truncated summation is taken to the left-hand side so that a homogeneous system results. The matrix organization used to represent these equations is given in Figure 4. Table 2 defines the computer variables noted in Figure 4, primarily for reference purposes in the next section.

The matrix that results is a function of the complex frequency s. A natural resonance occurs when s has a value such that the matrix has a zero determinant; hence, the homogeneous system of equations has a non-trivial solution. The next section explores some algorithmic considerations in the efficient generation and manipulation of the matrix.

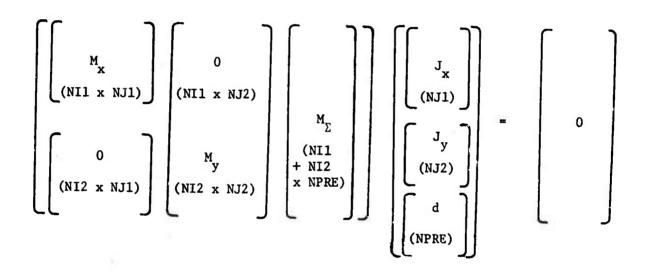


Figure 4. Organization of the System of Linear Equations

## Table 2

#### MATRIX FORMULATION PARAMETERS

NII Number of match points on the zoned quadrant of the plate.

NI2 = NI1

NPREJ Number of patches along the |x| = L/2 edge where  $J_x$ 

is preassigned to zero.

NPREI Number of patches along the |y| = w/2 edge where  $J_{y}$ 

is preassigned to zero.

NJ1 = NI1-NPREJ Number of unknown current values in  $J_{v}$  expansion.

NJ2 = NI2-NPREI Number of unknown current values in  $J_v$  expansion.

NJ3 NPRE = NPREI-NPREJ Number of preassigned current values (Also the number of coefficients retained in summation).

#### SECTION V

# ALGORITHMIC CONSIDERATIONS IN EVALUATING THE SYSTEM DETERMINANT

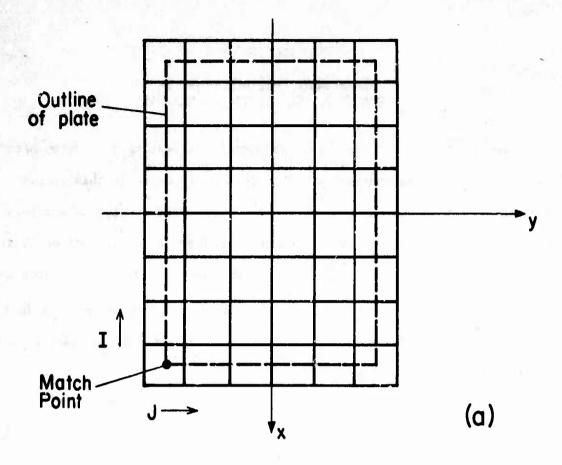
Some considerations taken into account in generating the system matrix and evaluating its determinant efficiently are discussed in this section. Since these two operations must be repeatedly carried out for many values of s in the course of determining the natural frequencies of the plate, it is essential that clean, efficient computer programming and coding be used so that execution of the program will be affordable. The volume of code in the algorithms is consistently compromised toward a larger size in order to meet the following two time-efficient objectives:

- 1. Avoidance of calculating the same quantity twice; and
- 2. Avoidance of logical decisions, particularly those which might be imbedded in loops.

The program is discussed in the context of the following major segments:

- Computation of an "interaction matrix";
- Construction of the non-zero submatrices of the system matrix from the interaction matrix;
- 3. Computation of the series terms' submatrix; and
- 4. Determinant evaluation.

The major contribution to the elimination of redundant calculations is the one-time computation of an "interaction matrix" which is made up of the individual kernel integral terms from (2) for all argument combinations which occur in the computation. The subsequent program step then picks, by subscript, entries from this matrix and constructs the appropriate kernel from one of equations (8) according to the symmetry conditions being solved. This procedure can be viewed in terms of the layout given in Figure 5a. The terms in the interaction matrix are those evaluated for the match-point as



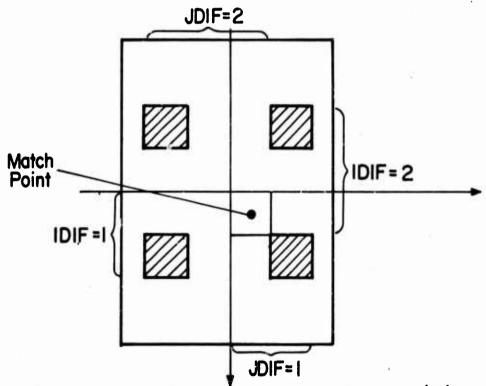


Figure 5. a) Conceptual Zoning for Calculation of the Interaction (b)
Matrix, b) Example of the Four Interaction Contributions
to a Single Source Term

shown in the lower left with the source patches indexed over the entire plate to generate the matrix. Thus, all geometric relationships which occur in the kernel terms are encompassed in the calculation. Note that all source patches are full patches for this calculation. The effect of half patches at the edges is accounted for by weighting by a factor of 1/2 the edge contributions. The kernel integral appropriate to the symmetry is constructed by summing with correct signs the appropriate elements from the matrix. Figure 5b gives an example of the four source patches entering into one kernel integral.

Differing degrees of sophistication are required in the calculation of the interaction terms depending on the spacing of the patches for which an interaction is being calculated. For the self patch, i.e., the patch in which the match-point resides, the integration of the kernel must be performed analytically because of the integrable singularity in the kernel there. Appendix A gives a series approximation to this integral. The result in Appendix A is evaluated directly in the program. For the patches adjacent to the patch containing the match point, the kernel is a rapidly varying but well-behaved function. The integration over these patches is evaluated numerically by a polynomial approximation. For patches further separated, the kernel is slowly varying and the integral is evaluated approximately as the product of the value of the kernel at the center of the patch and the area of the patch.

Some minor time economy is achieved in the matrix filling algorithm, which is a four-dimensional loop. The economy is found in the form of decision-free indexing, that is, the source contributions from interior patches, from |x| = L/2 edge patches, from |y| = w/2 edge patches, and from corners take on different forms. Rather than index over all source patches

with logical decisions implemented to discriminate among the four cases above, four different loops are used.

The computation of the series term submatrix is relatively straightforward. Because the Bessel-trigonometric products appear in two terms each,
they are all precalculated and stored in a vector. The individual terms are
then constructed from them.

The determinant evaluation profits significantly from an exploitation of the sparceness of the matrix. Either of two approaches may be taken to the sparce matrix manipulations. One is to separate the matrix algebraically and calculate an inverse as a composite of inverses of terms involving the submatrices. The alternative approach is to attack the matrix directly with Gaussian elimination, and to exploit the sparceness directly in the algorithm. The latter approach was chosen for the present purpose because it is judged to be slightly faster computationally and because in order to determine natural mode solutions for the SEM formulation, the homogeneous system of equations occurring at a pole must be backsolved. The algorithm resulting from the second approach is described in Appendix B.

The determinant evaluation routine itself appears in Appendix C as the function routine CPLATE.

#### SECTION VI

#### NUMERICAL CHECKS ON THE ACCURACY OF THE POLES

The results of some numerical checks on the accuracy of the pole location as determined from the numerical model described in Sections II through V are reported. It is shown that the model predicts well the poles for narrow strips possessing essentially thin scatterer resonance properties. Difficulties occur, however, in obtaining self-consistent results under different zone sizes for plates with larger aspect ratios. It is conjectured that the difficulty occurs because the subsectionally constant current representation is inadequate to represent the correct edge behavior for the currents-particularly the singular behavior for the parallel component. The rationale behind this conjecture is discussed.

Initial tests on the accuracy of the model were made for a rectangular strip with a shape ratio w/L = 1/10. Such a strip has an approximate equivalent dipole whose diameter-to-length ratio is  $1/10\pi$ .

Figure 6 gives the results of pole determinations for the first two poles for various numbers of pulses in the expansion of the current and for two different treatments of the edge pulse. The strip was zoned with one pulse across a quadrant. The numbers indicated in the figure are the numbers of pulses along the longitudinal direction of a quadrant. The "half-pulse" results are those obtained by the zone scheme described in Section IV where half-width pulses are placed at the edge so that match points fall at the edge. The "full-pulse" results are those obtained by zoning the plate with equal-sized pulses over the entire quadrant. In the latter case an a posteriori adjustment is made in the data correcting the length of the strip such that the end of the corrected strip lies at the end match-point.

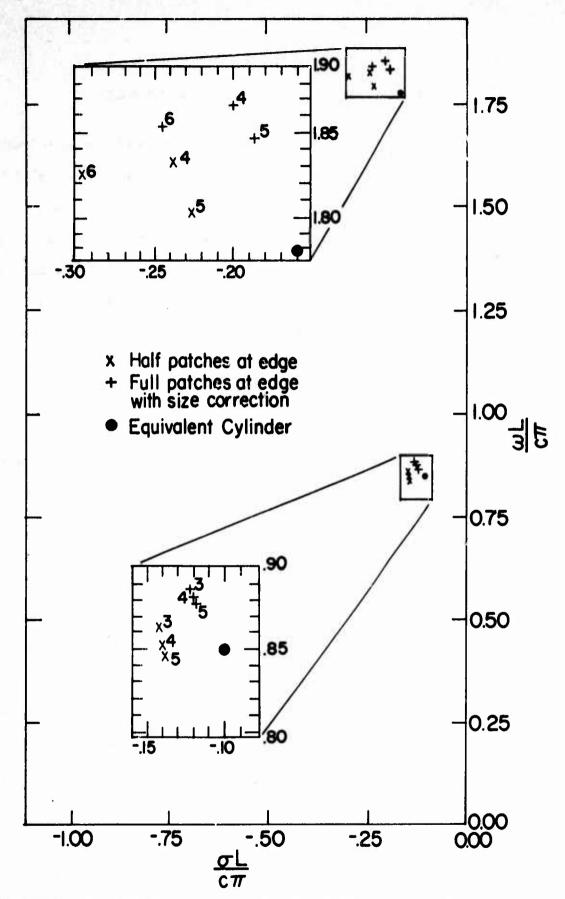


Figure 6. Calculated Pole Locations for Thin-Strip for Varying Numbers of Zones in the x-Direction and Different Edge Treatments (Cylinder Results from Ref. 6)

It is seen that the differences are small both for varying order and increasing pulse density. The NX = 6 results for the second pole show some departure from the trend established by the results for NX = 4 and NX = 5.

This is attributable to the fact that the matrix is on the brink of numerical instability for NX = 6. The results for NX = 7, which are not shown, are observed to be meaningless because of the instability manifested.

For comparison purposes, the first two poles for an equivalent cylinder (one whose circumference equals the strip width) are given as found in ref. 7. These results are judged reliable inasmuch as they have been cross-checked among several integral equation formulations and for several method-of-moments schemes. The equivalent radius taken is, of course, an approximation. It is seen that the half-pulse solutions compare slightly more favorably with the cylinder results. Because of this, and moreover, because the a posteriori data adjustment is avoided with the half-pulse scheme, it was used consistently in the remaining solutions.

The stable convergence properties of the numerical model exhibited for the thin-strip solution are not manifested for higher aspect ratios. The reason for the difference is that the strip is essentially a one-dimensional problem and the transverse (y-directed) component of current has little effect on the dominant longitudinal current. For wider structures the coupling is significant and inadequacies in the modeling of the singularities of the currents produce inaccuracies which are highly sensitive to zoning.

Figure 7 shows the results obtained for a pole trajectory as a function of the shape factor  $_{\rm w/L}$  where the zoning in the y-direction was adjusted

<sup>7.</sup> Umashankar, K. R., "The Calculation of Electromagnetic Transient Currents on Thin Perfectly Conducting Bodies Using the Singularity Expansion Method," Ph.D. Thesis, University of Mississippi, pp. 33-34, August 1974, (See also Tesche, F. M., IEEE Trans., Vol. AP-21, No. 1, pp. 53-62, 1972.)

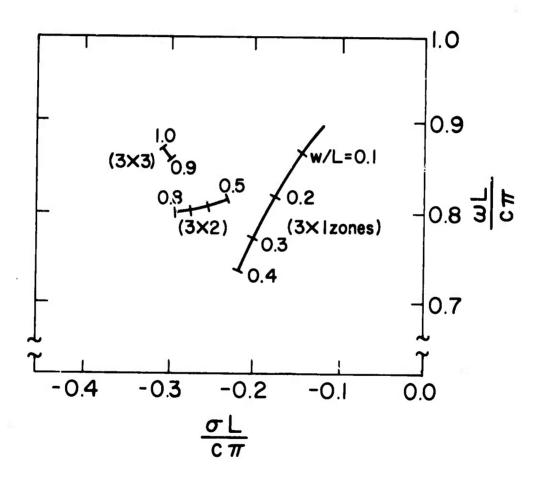


Figure 7. Computer Pole Trajectory Under Varying w/L with Zoning Changes

according to the value of w. It is evident that the solutions are unstable with respect to the zoning on the plate. Attempts to increase the number of zones significantly to improve upon the situation resulted in numerical instabilities in the matrix which cause the pole search iteration to fail.

The reason for the difficulty manifested in Figure 6 is believed to lie in the way that the edge of the plate is defined with the piecewise constant current expansion. Consider the characteristics of the two current components along a line traversing the plate in the y-direction as shown in Figure 8. The correct edge behavior at |y| = w/2 is that given in equations (3). The zoning scheme, however, forces  $J_X(x,\pm w/2)$  to take a finite value. The current extrapolates to a singular point for some y > w/2, i.e., the numerical model represents a plate whose width is greater than w.

If the number of transverse zones is increased as indicated by the dashed curve in Figure 8, the steepness of the edge behavior of  $J_{\rm X}$  is increased, and the extrapolation is characteristic of a narrower plate as compared to the first case. This narrowing of the effective width in the model is reflected in an increased Q (resonance quality factor) as the jumps in Figure 7 indicate.

One is tempted to conclude that a full-width pulse at the edge is a cure for the problem since the point at which the pulse current is defined is shifted relative to the edge as zoning is changed with full-width pulses. The effect of this procedure is to transfer the problem from component of current whose behavior is singular at the edge to the component which goes to zero. With full pulses at the edges, the normal component of current would go to zero interior to the plate rather than at the edge as it properly should.

An approach which is potentially a remedy for this difficulty is discussed in the conclusions.

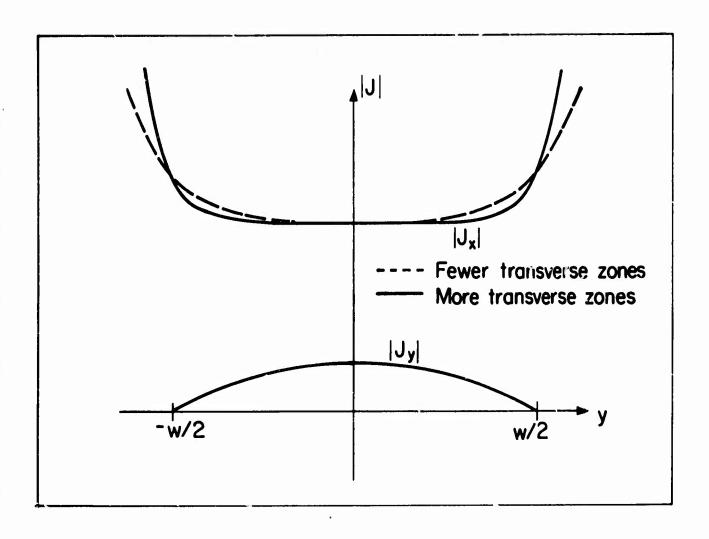


Figure 8. Behavior of Singular Component of Current at the Edge Under Change in Transverse Zoning

#### SECTION VII

#### POLE TRAJECTORIES AS A FUNCTION OF SHAPE RATIO

Figure 9 gives the results obtained for pole location for the lowest order pole of each of the symmetries as a function of w/L. Clearly, as the previous section indicates, the results cannot be taken as the correct results for the plate. However, the zoning was fixed for all w/L and the results are expected to reflect the proper trends for these trajectories.

The ++ and +- modes are in essence dipole modes, and their poles show the general lowering of Q as w/L increases. (The ++ indicates that the  $J_X$  component is symmetric both with respect to the x and y axes - see Table I.) The -- and can be thought of as a dipole mode in the transverse direction. As a result if shows a strong frequency dependence on the transverse dimension w. When w/L = 1, the -- mode is identical to the ++ mode rotated spatially 90 degrees. Consequently, the two trajectories coalesce as w/L  $\rightarrow$  1, when the zoning is 5x5 so as to preserve symmetry in the numerical mode. The failure of the 5x3 zone case is due to the reasons outlined in the previous section.

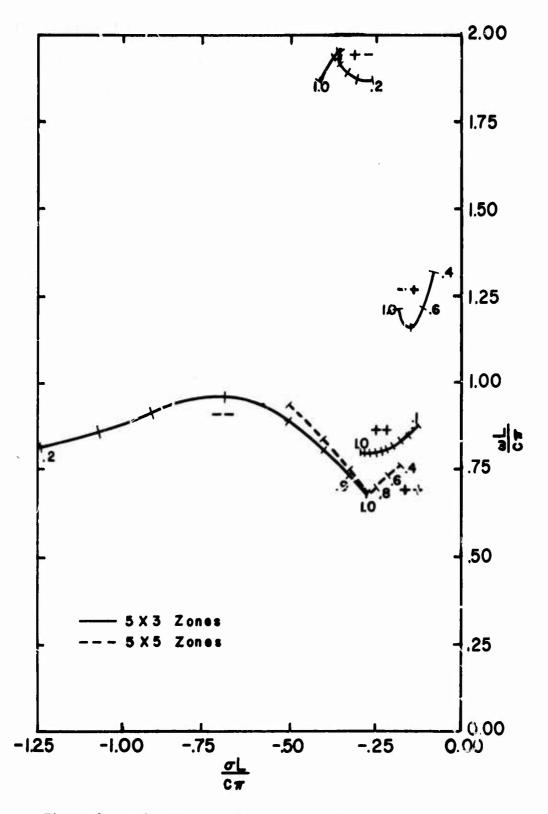


Figure 9. Pole Trajectories as Computed with Zoning Fixed

### SECTION VIII

#### CONCLUSIONS

The application of SEM to the equivalent problems of the perfectly conducting rectangular plate and the rectangular aperture in a perfectly conducting screen has been conducted with partial success. The applicability of SEM and the computational feasibility of determining SEM quantities are demonstrated. It is further demonstrated that by careful program construction, the computational costs of numerical treatment of two-dimensional problems can be made quite reasonable. The cost of generating a matrix and calculating its determinant by the methods discussed herein is less than 10 cents for each value of s.

Difficulties arise in the subsectionally constant current zoning because of a failure to properly model the edge conditions. Whereas Rahmat-Samii and Mittra (ref. 3) obtained good radar cross-section results with such a zoning scheme, the pole locations are highly sensitive to the zoning. Radar cross-section is a far-field quantity, and the integrated effects of the errors are small there. The pole locations, on the other hand, depend strongly on the dimensions of the structure, and it is evident that the plate size must be brought to bear in a precise fashion for the pole locations to be correct.

The edge problem can be remedied by imposing the edge conditions

(3) directly in the basis set elements for edge zones.

Davis has done this successfully for the circumferential component of current on a thick cylindrical scatterer (ref. 8). The circumferential current

Davis, W. A., "Numerical Solutions to the Problem of Electromagnetic Radiation and Scattering by a Finite Cylinder," Ph.D. Thesis, University of Illinois, 1974.

component is singular at the ends of the cylinder. The introduction of the singular basis element will produce a significant complication to the problem in that a second singularity, that of the current, must be integrated analytically in order to implement the model with edge conditions imposed. An independent check on program accuracy is dictated for a problem of this scope before proceeding with the edge condition approach.

## APPENDIX A

#### THE SELF-PATCH INTEGRATION

The term for the interaction matrix for IDIF = JDIF = 0, i.e., where the match point lies at the center of the source patch, can be written

$$I_{s} = 4 \int_{0}^{\Delta x/2} \int_{0}^{\Delta y/2} K(0,0|x',y') dx' dy'$$
 (A1)

This presumes a unit amplitude expansion pulse over the patch whose dimensions are  $\Delta x$  and  $\Delta y$ . The symmetry of the kernel with respect to both x' and y' is employed in the forming of (Al). This integral can be transformed to polar coordinates as

$$I_{s} = 4 \begin{cases} \tan^{-1} \frac{\Delta y}{\Delta x} & \frac{\Delta x}{2 \cos \phi} \\ \phi = 0 & \rho = 0 \end{cases} = \exp[-s\rho/c] d\rho d\phi$$

$$+ \int_{\phi = \tan^{-1} \frac{\Delta y}{\Delta x}}^{\pi/2} \int_{\rho = 0}^{\pi/2} \exp[-s\rho/c] d\rho d\phi$$

$$= -\frac{4c}{s} \begin{cases} \tan^{-1} \frac{\Delta y}{\Delta x} \\ \phi = 0 \end{cases} [\exp(-s\Delta x \sec \phi/2c) - 1] d\phi$$

$$+ \int_{\phi = \tan^{-1} \frac{\Delta y}{\Delta x}}^{\pi/2} [\exp(-s\Delta y \csc \phi/2c) - 1] d\phi \end{cases}$$
(A2)

If the exponential functions in the integrand are then expanded in McLauren series, the resulting terms of powers of secants and cosecants possess tabulated integrals. The result for three terms retained in the series is

$$I_{s} \approx -\frac{4c}{s} \left\{ -\frac{s\Delta x}{2c} \cdot 1/2 \ln q_{y} + 1/2 \left( \frac{s\Delta x}{2c} \right)^{2} \frac{\Delta y}{\Delta x} - \frac{1}{6} \left( \frac{s\Delta x}{2c} \right)^{3} \frac{\Delta x (\Delta x^{2} + \Delta y^{2})^{1/2}}{2\Delta y^{2}} - \frac{s\Delta y}{2c} \frac{1/2 \ln q_{x}}{2} + 1/2 \left( \frac{s\Delta y}{2c} \right)^{2} \frac{\Delta x}{\Delta y} - 1/6 \left( \frac{s\Delta y}{2c} \right)^{3} \frac{\Delta y (\Delta x^{2} + \Delta y^{2})^{1/2}}{2\Delta y^{2}} \right\}$$
(A3)

where

$$q_{\begin{pmatrix} x \\ y \end{pmatrix}} = \frac{\left[ (\Delta x^2 + \Delta y^2)^{1/2} + {\Delta x \choose \Delta y} \right]}{\left[ (\Delta x^2 + \Delta y^2)^{1/2} - {\Delta x \choose \Delta y} \right]}$$

## APPENDIX B

#### THE SPARSE MATRIX ALGORITHMS

### 1. Introduction

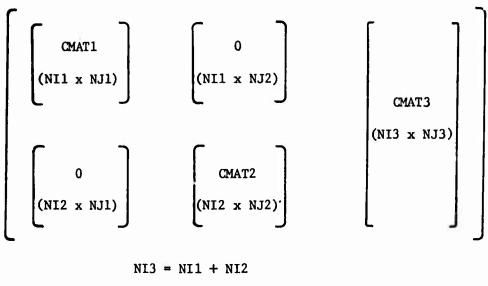
This Appendix describes the algorithmic approach to minimize the computation time involved in Gaussian elimination triangularization of systems of matrix equations which are "sparsely coupled." The term "sparsely coupled" as applied in this Appendix implies the matrix equation form given in (B1).

$$[M] [X] = \begin{bmatrix} M_1 & 0 \\ & & M_3 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} X \\ \end{bmatrix} = \begin{bmatrix} B \\ \end{bmatrix}$$
(B1)

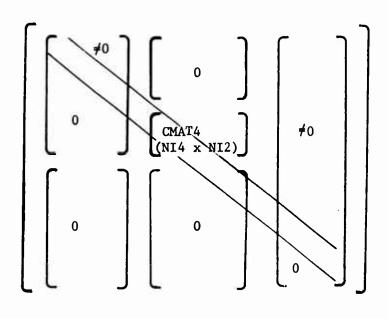
It is clear that in this form the sole coupling between the "upper" and "lower" systems of equations is contained in the matrix  $\mathbf{M}_2$ . Generally, the number of columns in  $\mathbf{M}_2$  is small compared with the order of the overall system.

An algebraic approach to exploiting the sparceness of (B1) results in solutions which are given in terms of the several submatrices and their inverses. (See, for example, ref. 9.) It is well-known, however, that it is sufficient for the purposes of determinant calculation and equation solution to triangularize the composite matrix in (B1). The triangularization process involves fewer operations than the diagonalization necessary for the calculation of an inverse.

<sup>9.</sup> Dunaway, O. C., "A Numerical Solution for the Distribution of Time-Harmonic Electromagnetic Fields in an Arbitrary Shaped Aperture in a Ground Screen," M.S. Thesis, University of Mississippi, 1974.



NI3 = NI1 + NI2 NJ3 = NI3 - NJ1 - NJ2(a)



NI4 = MAX (NI1 - NJ1,0)(b)

Figure B1. Submatrix Organization for the Sparse Matrix Algorithms, a) the Original Matrix, and b) Triangularized Form with the Generated CMAT4

This Appendix describes an algorithmic exploitation of the sparseness of the composite matrix in (B1). That is, a numerical process is given whereby only the non-zero subelements are stored and operated on, with the computational operations being those which render the composite matrix M upper triangular. The determinant of the composite matrix results directly from this triangularization. A solution for X in (B1) requires a backsolving process involving the triangularized form of M and a vector resulting from applying the elimination operations to the vector B. Routines to perform these operations are given as well.

Appendix C gives listings of the routines built on this algorithm. The triangularization routine is named SPRHOM. The backsolving procedure is performed by the entry HOMSLV to the routine SPRSLV. (The name entry SPRSLV backsolves an inhomogeneous system and is not used for present purposes.)

## 2. The Algorithm

The routine SPRHOM is simply an implementation of the Gaussian elimination procedure with maximum pivot selection applied to the composite matrix M in (Bl) viewed as a whole. The sparseness of M is exploited by storing only the non-zero submatrices in (Bl) and skipping the operations involving zero elements. The result is a substantial saving in both storage and computation time.

The forms of the input and of the end product for the triangularization are given in Figure (B1) with the sizes of the respective submatrices defined. It is recalled that the fundamental process in the Gaussian elimination procedure is the elimination of all or part of the elements of a column of a matrix with respect to a pivot element, commonly the element of greatest magnitude in the column. That is, for a column under process, the row

containing the main diagonal element of the matrix which falls in that column. All or part of the elements not on the main diagonal are "eliminated" or made zero by subtraction of some multiple of the row containing the column maximum. In the triangularization procedure, the part of the column comprising elements lying below the main diagonal after row exchange are eliminated. If the matrix is a part of a system of equations with non-zero right-hand side, the row operations of exchange and subtraction of a constant multiple of another row must be performed on the corresponding elements of the right-hand side vector as well.

The algorithm of the routine SPRHOM operates according to the Gaussian elimination procedure described above. However, the three submatrices CMAT1, CMAT2, and CMAT3 are stored individually. In addition, the routine generates a submatrix CMAT4 in the course of selecting pivots for the columns contained in CMAT2. Further, the "elimination" of elements of submatrices that are zero a priori is skipped. The result is significant storage and time economy.

In order to minimize logic decisions in the routine, it is organized to operate sequentially through the partitioned matrix. The steps are as follows (it is convenient to follow the thinking of these steps by tracing the location diagonal of the composite with the aid of Table Bl):

- a. Perform conventional Gaussian elimination to zero the elements CMAT1(I,J) for I > J, i.e., the elements below the main diagonal of M. Choose maximum pivot elements in conventional fashion. Carry row operations into CMAT3.
- b. Create CMAT4 by row swapping with CMAT2 so as to choose maximum pivot elements. Perform elimination to zero CMAT4 elements for I > J and the entire column of CMAT2. The number of rows created in CMAT2 is NI4 = NI1 - NJ1, the amount by which CMAT1 is over-square. Carry row operations across into CMAT3.

- c. Choose maximum pivot rows in columns of CMAT2 with J > NI4 and swap with rows given by I = J - NI4 (the rows containing the Jth column diagonal element of the composite). Conduct elimination to zero elements with I > J + NI4. Carry row operations into CMAT3.
- d. Conduct conventional pivot selection and elimination on CMAT3 to zero elements CMAT3(I, J) with I > J + NJ1 + NJ2.

In each column elimination operation, the pivot value is multiplied into a product accumulator to produce a value for the determinant of the composite matrix. The sign of this product is changed at each row swap in accord with the rates of matrix algebra row operations.

The backsolving routine SPRSLV with its entry HOMSLV operate in a straightforward manner on the submatrices as reduced by SPRHOM.

Details are omitted here, but the routines may be easily followed in a row-by-row flow from the bottom of the composite matrix, if one keeps in mind the row index relations of column 4 of Table Bl. The entry HOMSLV assumes a zero determinant value resulted (approximately) from SPRHOM and the last element of the solution vector is picked as unity. (The zero determinant results from a zero falling at the last diagonal location in maximum pivoting Gaussian elimination.) The remainder of the homogeneous solution vector is backsolved conventionally relative to this last element. The vector is then renormalized so that its maximum element is unity.

Table B1

PRIMARY INDEXING QUANTITIES IN THE ALGORITHM

Submatrix	Size of <sup>1</sup> Submatrix	Indices of Main 1 Diag. of Compos.	Relative Row <sup>2</sup> Index of CMAT3 and CRHS
CMAT1	NI1 x NI2	(J,J)	I3 = I
CMAT4	NII - NJI x NJ2 (can be null)	(J,J)	I3 = I + NJ1
CMAT2	NI2 x NJ2	(J - (NI1 - NJ1), J)	13 = 1 + N11
CMAT3	NI1 + NI2 x NI1 + NI2 - NJ1 - NJ2	(J + NJ1 + NJ2, J)	13 = 13

- 1. Quantities given in terms of input parms. to the routine. Related internal quantities are given in Figure Bl.
- 2. Relative to I, the row index of the submatrix in question.

# APPENDIX C

# PROGRAM LISTINGS

All code compileable on ISM OS/360 and OS/370 FORTRAN levels G or H.

The routine ZANLYT and its service routine UERTST is taken from the program
library FORTUOI made available by the Computer Services Office, University of
Illinois at Urbana-Champaign, Urbana, Illinois 61801. The routines BSLJZ
and BSCJZ are taken from the International Mathematical and Statistical
Library (IMSL). They may not be reproduced apart from this application
program package. The IMSL library is available by subscription from IMSL,
Inc., 6100 Hillcroft, Suite 510, Houston, Texas 77036.

```
POLE SEARCH PROGRAM FOR S E M FORMULATION OF THIN-PLATE SCATTERER
                                                                               00010
      BY L W PEARSON 8/74
                                                                               00020
                                                                               22230
      IMPLICIT REAL+8(A,R,D-H,C-Z),COMPLEX+16(C)
                                                                               00040
      COMMON /GEDM/ XSYM. YSYM. W.Y. NY. 1PPEAS(10), JPREAS(10), NPREI, NPREJ
                                                                               00050
      INTEGER MES(4,2)/'SYMM', 'ETRT', 'C', ' ', 'ANTI', 'SYMM', 'ETRI', 'C'/
                                                                               22160
      DATA C/(3.DO8,O.DO)/,PLUS/++/,PI/3.141592653589793/
                                                                               00070
      DATA HX/'X'/, HY/'Y'/
                                                                               00080
      EXTERNAL CPLATE
                                                                               20090
      DIMENSION CX(20) . INFEP(20)
                                                                               00100
      LOGICAL LAUTO
                                                                               00110
      READ(5,1,END=999) XSYM,YSYM,NX,NY,WO, WS,WM,CSUNCP,LAUTO
                                                                               20120
      FORMAT(281,2X,213,5F10.4,T80,L1)
                                                                               00130
                                                                               20140
      [ MX =1
      IMY=1
                                                                               201 50
      TE(XSYM.NE.PLUS) TMX=2
                                                                              00160
      TE(YCYM.NE.PLUS) JMY=2
                                                                               20170
      YW= (WM-WO)/WS
                                                                               20180
      TE(N#.GT.0) GO TO 105
                                                                               00190
      NW=-NW
                                                                              00200
      WS=-WS
                                                                              00210
      TE(WS#NW.LT.WM-WT) NW=NW+1
 105
                                                                               10220
      DO 200 IW=1.NW
                                                                              20230
      H=WP+([W-1]*WS
                                                                              00240
      IF( .NOT .LAUTO) GO TO 140
                                                                              00250
                                         SKIP PAST AUTO ZONING
                                                                              00260
0
                                                                              20270
      ROUTINE TO DETERMINE NO OF EXPANSION PULSES BASED ON ELECTRICAL
                                                                              00280
r
      SIZE OF PLATE
                                                                              0.02.90
C
                                                                              00300
      TESTWV= .1.885010/DARS(DTMAG("SUNCP))
                                                                              20310
                                                                              20320
ŗ
      11111111111111
                                                                              00330
C
                                                                              00340
      NPPWVL=6
                                                                              00350
                                                                              00360
Ċ,
      00370
                                                                              103.80
      FLENX=1/TESTWV
                                                                              00390
      NX=IDINT(FLFNX*NPPHVL)
                                                                              00400
      IF(DFLOAT(NX).LT.FLENX+NPPWVL) NX=NX+1
                                                                              22412
      FLFNY=W/TESTWV
                                                                              00420
      YY= IDINT (FLENY+NPPWVL)
                                                                              00430
      [F(DFLOAT(NY).LT.FLFNY*NPPWVL] NY=NY+1
                                                                              00440
      NX=MINO(NX,5)
                                                                              00450
      NY=MINO(NY,5)
                                                                              20460
                                                                              00470
•
      BEGIN SETUP FOR ALTERNATE EDGE PATCH PREASSIGNMENT
                                                                              00480
Ç
                                                                              22492
 140
     NPPEI = (NX+2)/3
                                                                              00500
      NPREJ = (NY+2)/3
                                                                              00513
      TF(NX-2*NPREI+2.LF.1.AND.MPPFT.GT.1) NPRFT=MPPEI-1
                                                                              10520
     IF(NY-2*NPREJ+2.LE.1.4ND.NPRFJ.GT.1) MPREJ=NPREJ-)
                                                                              00530
      D? 110 I=1. NPRET
                                                                              22540
     TPREAS(NPREI+1-T)=NX-3+T+3
                                                                              00550
 110 CONTINUE
                                                                              01560
      DO 120 J=1, NPREJ
                                                                              00570
      JPREAS(NPREJ+1-J)=NY-3*J+3
                                                                              00580
120
     CONTINUE
                                                                              12592
                                        LOCATIONS WHERE X-DIRECTED CUPPEN
                                                                              00600
                                        T IS SET TO ZERO GIVEN BY SUBSCRI
                                                                              00610
```

۲	PTS (NX. JPREAS) AND Y-DIRECTED BY	00620
r	(IPFEAS, NY)	00630
c	ATTE CAS PIATE	00640
C	UDITE/4 23 H CCINOD	20650
•	WPITE(6,2) W.CSUNOP	
2	FORMAT('IENTER ITERATION', /, 'OSHAPE RATIO =', F5.3,5X,	00660
	1'STARTING FREG = ', 2012.4)	20670
	WRITE(6,3)	00680
3	FORMAT( *O', 10 %, 'CUR SYMMETRY', 6%, 'PULSES', 3%, 'PREASSIAN LOC''NS')	00690
	WRITE(6,4) HX,(MES(T,IMX),T=1,4),NX,(TPREAS(J),J=1,NPREI)	00700
4	FORMAT(* ', A1, '-DIR', 5X, 444, 16, 5X, 10(3)	0071.0
	WPTTE(6.4) HY.(MES(I.IMY).;=1.4).NY.(JPPEAS(J).J=1.NPPEJ)	00720
	WRITF(6,5)	22730
5	FORMAT('0',11X,'COMPLEX FREQ',17X,'DETERMINANT')	00740
	CX(1)=CSUNPR	00750
	CALL ZANLYT (CPL 4TE, 1.D-50, 4, 0, 1, 1, CX, 100, INFER, IFF)	00760
	WRITE(6,6) (X(1)	00770
6	FQPMAT('ORETURN FROM ITERATION',/,'OPCLE LOC''N', 2E12.4)	00780
	CALL MODE	00790
	CSUNDP=CX(1)	00800
200	CONTINUE	00810
	93 T0 199	00820
999	O STIP	00830
	END	00840



```
SUBPRUTINE MODE
                                                                                0085C
     IMPLICIT REAL+8(A.R.D-H.C-Z), COMPLEX+16(C)
                                                                                00860
     DAMON /MAT/ CMATX(25,25), CMATY(25,28), CHOM(50,10), CMAT4(10,25).
                                                                                0087C
    IMPTCHS, NDIMI, NDIMCI, NDIMCJ, NOPO
                                                                                00880
     COMMON /GEOM/ XSYM.YSYM.W.NX.NY.IPRFAS(20).JPPEAS(10).NPREI.NPRFJ
                                                                                00890
     DIMENSION PPRX(5,5), CPRY(5,5)
                                                                                00900
     DIMENSION CJ (50)
                                                                                00910
     NPRE-NPREI+NPREJ
                                                                                00920
     NPREIM=NPREI-1
                                                                                00930
     NPREJM=NPREJ-1
                                                                                00940
     CALL HOMSEVIC MATX, NPTCHS, NPTCHS-NPREJ, NEIM1, NDIM1,
                                                                                00950
                  CMATY. NPTCHS. NPTCHS-NPREI. NOIM1. NDIM1.
    1
                                                                                00960
                  CHOM, NDINCI, NOIMCJ, CMAT4, NDIMCJ, NDIMI, CJ, NORD)
                                                                                00970
     NXM1=NX-1
                                                                                00980
     NYM1=NY-1
                                                                                00990
     NSUP S=C
                                                                                01000
     07 470 JS=1.NY
                                                                                01010
     DC 450 IS=1.NXM1
                                                                                01020
                                                                                01030
     J=(JS-1)+NX+TS
     CPRX(IS, JS) = CJ(J-NSUBS)
                                                                                01040
     JM=J-NSUBS
                                                                                01050
     CONTINUE
                                                                                01060
                                                                                01070
     J=JS*NX
     IF(JS.NE.JPREAS (NSUBS+11) GO TO 460
                                                                                01080
     NSUPS = MINO(NSUBS+1.NPREJM)
                                                                                01090
     CPRX(VX.JS)=(0..0.)
                                                                                01100
     GD TD 470
                                                                                01110
     CPRX(NX,JS) =CJ(J-NSUBS)
                                                                                01120
460
470
     CONTINUE
                                                                                01130
     DO 500 TS=1,NX
                                                                                0114C
                                                                                01150
     DO 500 JS=1, NYM1
     J=(JS-1) + VX+ TS
                                                                                01160
     CPRY(IS, JS) = CJ(NPTCHS-NPPEJ+J)
                                                                                01170
     CONTINUE
                                                                                01180
     VS:JPS=0
                                                                               01190
     DO 530 IS=1.NX
                                                                                01200
     J=NYM1+NX+TS
                                                                                01210
     IF(TS.NF.IPREAS(NSURS+1)) GO TO 510
                                                                                01220
     SPRY(15,NY)=(0.,0.)
                                                                                01230
     VSUBS=MINO(VSUBS+1,NPFEIM)
                                                                                01240
     30 TO 530
                                                                                01250
510 CPRY(IS,NY) = CJ(NPTCHS-NPPEJ+J-NSURG)
                                                                                01260
    CONTINUE
                                                                               01270
530
     WRITE(6.16)
                                                                                01280
     FORMAT( *O++++NATURAL MODE+*+** * *, /, *OX-DIRECTED CHERENT*)
                                                                                01290
16
                                                                               01300
     DO 540 T=1. NX
     WPITE(6,17) (CPRX(I,J),J=1,NY)
                                                                               01310
     F7PMAT( '0', 5( 2012.4,2X))
                                                                               01320
17
    CONTINUE
                                                                                01330
540
     WP! TE (6,18)
                                                                               01340
     FORMAT( 'OY-DIRECTED CURRENT')
                                                                               01350
     DO 550 X=1. NX
                                                                                21360
     WRITE(6,17) (CPPY(I,J),J=1,NY)
                                                                               01370
     CONTINUE
                                                                               01380
     WP ITE(6,19)
                                                                                21392
     FORMAT( OHOMOGENEOUS EXPANSION COEF'IS!)
                                                                               01400
     WRITE(6,17) (CJ(2*NPTCHS-NPRE+I), I=1, NPRE)
                                                                               01410
     PETURN
                                                                               01420
                                                                               01430
     EVD
```

```
COMPLEX FUNCTION CPL 4TE+16 (CSUNDR)
                                                                                 01440
       DETERMINANT EVALUATION POUTING FOR HALLEN-TYPE AUGMENTED MOMENT
                                                                                 01450
       MATRIX FOR THE THIN PLATE SCATTERER
                                                                                  01460
C
       FOR S E M APPLICATIONS
                                                                                  21470
      BY L W PEARSON 8/74
                                                                                 01480
                                                                                 01490
       IMPLICIT COMPLEX*16(C), FEAL*8(A,R,D-H,C-Z)
                                                                                 01500
       COMMON /GEOM/ XSYM.YSYM.W.NX.NY.IPPEAS(10).JPPEAS(10).NPREI.NPREJ
                                                                                 01510
      COMMON /MAT/ CMATX(25,25), CMATY(25,25), CHCM(50,10), CMAT4(10,25),
                                                                                 01520
     INPTCHS, NOIMI, NOIMCT, NOIMCJ, NORD
                                                                                 01530
      REAL+8 DRARG, DIMAPG, DRRES(20), DIMBES(20), DUM1(20), DUM2(20), DUM3(20
                                                                                 01540
     11,0044(20)
                                                                                 C1550
       DIMENSION CINTER (10,10), CINTX(25), CÎNTY(25), CCOSTM(10), CSINTM(10)
                                                                                 01560
       INTEGER MES(4,2)/'SYMM', 'FTRI', 'C', ' ', 'ANTI', 'SYMM', 'ETPI', 'C'/
                                                                                 01570
       DATA C/(3.D08,0.D0)/, PLUS/!+1/,PJ/3.141592653589793/
                                                                                 11580
      NOTM1 = 25
                                                                                 01590
      NOIMC 1=50
                                                                                 01600
      NDTMCJ=10
                                                                                 01610
      VDTM=50
                                                                                 01620
                                                                                 01630
ſ,
      FORMULATION SETUP ROUTINES
                                                                                 01640
                                                                                 01650
      TYX=1
                                                                                 01660
      I MY=1
                                                                                 01670
      IF(XSYM.NE.PLUS) TMX=2
                                                                                 01680
      IF(YSYM.NF.PLUS) TMY=2
                                                                                 01690
                                          IM(X/Y)=2 INDICATES ENTISYMMETRIC
                                                                                 01700
                                          DISTR OF X-DIRECTED CURRENT WPT X
                                                                                 01710
                                          /Y AXIS
                                                                                 01720
      VDTC-15=NX+NY
                                                                                 01730
C
                                          TET NO OF CUPRENT PATCHES
                                                                                 01740
                                                                                 01750
      NXM1 = NX - 1
      MYM1 =NY-1
                                                                                 01760
      EDGEAC=0..5
                                                                                 01770
                                          PROPORTIONAL WIDTH OF EDGE PULSES
                                                                                 01780
      EDG 2= EDGFAC + EDGFAC
                                                                                 01790
                                          COPNER FACTOR
                                                                                 01800
      DX=1./(FLOAT(2+MX-2)+2+FOGFAC)
                                                                                 01810
      DY=W/(FLD&T(2*MY-2)+2*EDGFAG)
                                                                                 01820
      VXT2=VX*2
                                                                                 01830
      NYT7=NY+2
                                                                                 01840
      CS=CSUNCP/2/7
                                                                                 01850
                                          NORMALIZED LAPLACE VAPIANLE
                                                                                 01860
      TNTPTS=13
                                                                                 01870
      DXINT = DX/12
                                                                                 01880
      DYINT=DY/12
                                                                                 01890
                                          NUMER INTEG PAFMS
                                                                                 01900
      XMY ** (I-)-=XMY2N
                                                                                 01910
      NSYMY = -(-1) * * IMY
                                                                                 01920
                                          SIGNED SYMMETRY INDICATORS
                                                                                 01930
      NC MT T = NSYMY
                                                                                 01940
      WYPH * XMYZH = JITMZH
                                                                                 01950
      NSMIN=NSAWX
                                                                                 01960
                                          SIGNS OF KERNEL FOR EA DUAD'S CON
                                                                                 01970
Ç
                                          TP TRUT TON
                                                                                 01980
      NINDX=2
                                                                                 01990
      TE(NSMIII.GT.O) NINDX=1
                                                                                 02000
                                          NINDX = 1 INDICATES EVEN-EVEN OR
Ċ
                                                                                 02010
C.
                                          COD-COD SYMMETRY FOR X-DIR CURP
                                                                                 02020
                                                                                 02030
      NSCCS=1
      IF(XSYM. FQ. PLUS) NSCC = 2
                                                                                 02040
```

```
C
                                         Y FOR X DIR CURR (I E COSINE EXPA
                                                                                05060
                                         NSION OF HOMGENEOUS SOLIN)
                                                                                02070
C
                                                                                22280
                                                                                02090
      NPRF=NPREI+NPREJ
                                         TOT NO OF PRESSIGNED CURP VALHS
C
                                                                                02100
      NPRFJM=NPRFJ-1
                                                                                02110
      NPREIM=NPREI-1
                                                                                02120
      YPREPI=NPRE+1
                                                                                02130
C
      END OF INPUT/SPECIFICATION ROUTINES
                                                                                92149
                                                                                02150
C
      ROUTINE TO FILL INTERACTION MATRIX FROM WHICH MOMENT MATRIX IS
                                                                                02160
C
      CONSTRUCTED
                                                                                021 70
                                                                                02180
      DIAG = DSORT ( DX +DX+DY+DY)
                                                                                02190
                                                                                JSSU0
      ALNXTM=2+DLOG((DI4G+DY)/DX)
                                                                                0?210
      ALNYTM=2+DLOG((DIAG+DX)/DY)
      DYBDX = DY/DX
                                                                                02223
                                                                                02230
      DXBDY=DX/DY
      CSDX=CS+DX
                                                                                02240
      CSDY=CS*DY
                                                                                02250
      CSDX2=CSDX+CSDX
                                                                                02260
      CSDX3=CSDX+CSDX2
                                                                                02270
      CSDX4=CSDX2+CSDX2
                                                                                228°
      CSDY2=CSDY+CSDY
                                                                                02290
                                                                                02300
      CSDY3=CSDY*CSDY2
                                                                                22310
      CSDY4=CSDY2*CSDY2
                                         COMPONENT TERMS FOR SELF-PATCH SE
                                                                                02330
C
                                         RIES
                                                                                02330
C
      CXTERM=-0.5D0 +CSDX+&LNXTM+0.5D0+CSDX2+DYPDX-CSDX3+LDXBDY+DJAG/(12+
                                                                                02340
            4LNXTM/24)+CSDX4+DVBDX+(1+DYBDX+DYBDX/3)/24
                                                                                32350
     10Y)+
      CYTERM=-0.500 *CSDY*ALNYTM+0.500*CSDY2*DXBDY-CSDY3*(DYPDX*DIAG/(12*
                                                                                02360
             ALNYTM/24) +CSDY4*DXRDY*(1+DXRDY*DXRDY/3)/24
                                                                                02370
                                         CALC INDIV SERIES FOR SELF-INTER
                                                                                02380
      CINTER(1,1)=-2/CS * (CXTERM+CYTERM)
                                                                                22390
                                         COMPUTE SELE-INTERACTION
C
                                                                                02400
      DO 220 IS=1,2
                                                                                02410
      XS=(FLOAT(IS)-1.500)+0X
                                                                                02420
      D7 220 JS=1,2
                                                                                02430
                                         LOOP TO CALC ADJACENT PATCH INTER
C
                                                                                02440
      IF(IS*JS.EQ.1) GO TO 220
                                                                                02450
      YS=(FLOAT(JS)-1.500) +DY
                                                                                02460
      DO 210 JENT=1, THTPTS
                                                                                02470
      XP=FLDAT(![NT-1)*DX[NT
                                                                                02480
                                         NUMER INT WRT X LOOP
                                                                                02490
      X2TM2=XS+XP
                                                                                22500
      X2TM2=X2TM2+X2TM2
                                                                                02510
      DO 200 JINT=1, INTPTS
                                                                                72520
      YP=FLOAT (JINT -1) + DY INT
                                                                                22530
                                         NUMER INT APT Y LOOP
                                                                               02540
      Y?TW=YS+YP
                                                                                02550
      P=DSOPT (X2TM2+Y2TM+Y2TM)
                                                                                02560
      TINTY(JINT) = CDE XP(-2*CS*R)/P
                                                                               02570
                                         EVAL INTEGRAND
                                                                               22580
      CONTINUE
200
                                                                               02590
      CALL DWEDDL(CINTY, INTPTS, DYINT, CINTX(IINT))
                                                                               02600
                                         INT WET Y TO YIELD X THIESPAND
                                                                                12610
     CONTINUE
                                                                               02620
      CALL DWEDDL(CINTX, INTPTS, DXINT, CINTER(IS, JS) >
                                                                               02630
                                         INT WPT X
                                                                               2640
220 SONTENUE
                                                                               02650
```

VSCOS = 2 INDICATES EVEN SYMM WRT

	00 250 IS=1,NXT2		72660
	X2TM2=DFLOAT(IS-L)+DX		02670
	X2TM?=X2TM2*X2TM2		02680
_	D7 250 JS=1,NYT2	1 0000 500 PENATNOED OF INTERACTIO	02690
C		LOOPS FOR REMAINDER OF INTERACTION	02700
ζ.	*******	N CALC'ED BY CHE-PT PECTANG PULE	
	IF(IS+JS-LT-4-OR-IS-FO-2-AND-J	15.69.21 GU 10 200	02720
	Y2TM=FL		02740
	REDSORT(X2TM2+Y2TM+Y2TM)	+0×+0×	02750
250	CINTER(IS,JS)=CDEXP(-2*CS*P)/P	**()X*U*	02760
250	- CONTINUE - END DE LOOP TO FILL INTERACTIO	MATRIX	02770
Č	END 15 CODE to SICO INTERSTATO	L WAIKIY	12780
C	BEGIN CONSTRUCTION OF MOMENT M	MATRI	02790
Ċ	Dentil 4 Cont 2: FOC LT 14 COS OF BUILDING B	T. L. I. V	02900
1,	D7 350 TM=1.NX		02810
	00 350 JM=1.NY		02820
C	93 390 SMATANA	INDEXING OF MATCH-POINTS OVER ENT	02830
Ċ		TRE QUACPANT	02840
١,	[=(JM-1)*NX+T*	(45 40) ( - 24)	02850
c	(-(54-1)-484)	ONE-DIM MATCH-PT INDEX GENEED	02860
r.		COLIMISE ALONG X-DIRECTION	22870
•	VSUPS=0	S. F. Make Merido X annual C. C. M.	02880
	DO 330 US=1 , NYM1		22890
S	53 330 33-CFMF-L	THEREX EVER SCHREE PATCHES Y-DIR	02900
10.0	JD1=[ABS(JM-JS)+1	THEEX TYEE STORES STORES TO BE	02910
٢	301-1403(3 - 33711	157 AND 2ND QUAD J IDIFFERENCE	02920
r		INDEX'	02930
	Jn2=JM+JS		02940
C		3RD & 4TH QUAD J *DIFFERENCE	02950
Č		TNDE X*	02960
ŕ		NOTE THAT "DIFFERENCE INDICES" AR	02970
C		F = 'INDEX DIFFERENCE' +1 FOR THE	02990
Č		SAKE OF FORTERN THOEXING	02993
	DO 310 IS=1, NXM1		03000
r.		INDEX OVER SOURCE DATCHES X-DIE	03010
	TD1=TABS(TS-TM)+1		03020
C		IST & 4TH QUAD "I JEE PEDEX"	3 3 430
	102=15+TM		03941
C		2ND & 300 OLIED + OTER INDEX+	03757
	J=(J S-1 ) *NX+ T S		03060
Ç		UNE-DIM SCHOLE-DI IMUEX	0.30.40
	CO=CINTER(ID1,JD1)+NSMITI*CINT		03080
Ċ		SUM DE SOURCE CONT EROM OF & GITT	03090
	CE=NSMIT *CIVTER (ID2.JD1)+MSMIV		03100
C		ZIM CE ZUNZCË CUMI EEGM DII E DIV	031 <b>1</b> 0
	CMATX(T.J-NSURS)=CO+CF		03120
r		SUPPAT ENTRY FOR X-DIG CUEPIS	03130
	$CMVLA(I \cdot I) = CU-CE$		13140
		SURMAT ENTRY FOR Y-DIG CUPPIS	03150
Ç		NOTE THAT EVEN O'S CONT MEGATIVE	03160
۲		FOR Y-DIR CUPPIS	03170
310	CONTINUE		03180
C		O DATCHEC	03190
C	END OF SOURCE LOOP FOR INTERIO	K NV CHEZ	03200
ŗ	CONCEDUCATION OF COURSE TERMS O	TEN ADELVI-A EDEE EDILOUE	03210
	CUMCIBULITUM OF SURBLE TERMS E	TUM STOLATES BUGE FULLIMS	13220
С	TOT-TARCENIA THE-		03236
	TD2=NY+IM		03240 03250
	TD2=NX+IM		
	J=JS+NX	t	03260

```
CD=CINTER(ID1.JD1)+NSMITT*CINTER(ID2.JD2)
                                                                                 03270
                                          SUM OF SOUPCE CONT FROM UI & QIII
                                                                                 23280
       SE=NSMTT#SIMTER(ID2,JD1)+MSMTV*CINTER(ID1,JD2)
                                                                                 03290
                                          SUM OF SCUPCE CONT FROM QII & OIV
                                                                                 03300
C
       CMATY(I.J)=(CD-CE)+FDGFAC
                                                                                 03310
                                          SURMAT ENTRY FOR Y-DIR CURR'S
                                                                                 03320
                                          NOTE THAT EVEN O'S CONT NEGATIVE
                                                                                 03330
                                          FOR Y-DIP CURRIS
                                                                                 03340
       IF(JS.NE.JPREAS(NSURS+1)) GO TO 325
                                                                                 03350
       NSUBS: MINO(NSUBS+1, NPREJM)
                                                                                 03360
       30 th 330
                                                                                 03370
 325
       CMAIX(I, J-VSURS)= {CE+CC) *FDGF6C
                                                                                 03380
                                          SUBMAT ENTRY FOR X-DIR CURR'S
                                                                                 03390
C
                                                                                 03400
•
       FND POUTINE FOR ABS(X)=+ SDGE TERMS
                                                                                 03410
                                                                                 03420
      CONTINUE
 330
                                                                                 03430
                                                                                 03440
Ç
       END LOOP OVER Y COOPD FOR INTERIOR PATCHES
                                                                                 9345C
                                                                                 03460
•
       REGIN ROUTINE FOR CONSTRUCTION OF SOURCE TERMS FOR ABS(Y) = B EDGE
                                                                                 03470
C
                                                                                 03480
       JD1 = TARS(NY-JM) + 1
                                                                                 03490
       ML+YV=SOL
                                                                                 03500
       NSIJRSJ=NSIJRS
                                                                                 03510
       VSUPS=0
                                                                                 03520
       D7 340 TS=1,NXM1
                                                                                 03530
C
                                          INDEX DOWN X FOOPD'S INTERIOR
                                                                                 03540
Ç
                                          DATCHES
                                                                                 03550
                                                                                 03560
       TO1 = TABS (TS-IM)+1
       TD2=15+14
                                                                                 03570
       2 T+XV* ( JMYN )=L
                                                                                 03580
      CO=CENTER(ID1, JD1)+MSMIII*CENTER(ID2, JD2)
                                                                                 13590
                                          SUM OF SOURCE CONY FROM OF & OTH
                                                                                 03600
      CE=NSMIT *CINTER(ID2, JD1) +NSMIV*CINTER(ID1, JD2)
                                                                                 03610
                                          SUM OF COURCE CONT FROM QII & DIV
                                                                                 23620
C
      CMATX(I.J-NSUBSJ)=(CE+CC) *FDGFAC
                                                                                 03630
                                          SUBMAT EMTRY FOR X-DIR CURPIS
                                                                                 03640
      TECTS.NE. [PREAS(NSUPS+1)) GO TO 335
                                                                                 03650
      VSUPS=MINO(NSUPS+1, NPRFIM)
                                                                                 03660
      GO TO 340
                                                                                 03670
       ^MATY(I,J-NSUBS)=(^^-CE)*EDGEAC
                                                                                 03680
 335
                                          SURMAT ENTRY FOR Y-DIR CUPRIS
                                                                                 03690
                                          NOTE THAT EVEN O'S CONT NEGATIVE
Ċ
                                                                                 03700
                                          FUD Y-DIR CHERIS
                                                                                 03710
      CONTINUE
                                                                                 03720
C
                                                                                 03730
      END POUTINE FOR ABS(Y) = R FOGE
                                                                                 03740
                                                                                 03750
C
      CONSTRUCTION OF CORNER SOURCE TERM
                                                                                 0376C
                                                                                 03770
      ID1=TABS(NX-IM)+1
                                                                                 03780
      TD2=MX+TM
                                                                                 03790
      YV+XV=L
                                                                                 03800
      CD=CINTER(ID1, JD1)+NSMITT*CINTER(ID2, JD2)
                                                                                 03810
                                          SUM OF SOURCE CONT FROM OI & OIII
                                                                                 03820
      CE=NSMII + INTER (ID2.JO1) +NSMIV#CINTER (ID1.JD2)
                                                                                 03930
                                          SUM OF SOURCE CONT FROM QII & DIV
                                                                                 03840
      TE(NY.NE.JPREAS(NPPEJ)) CMATX(T.J-NPREJM) = (CE+CD) * CDG?
                                                                                 03850
                                          SURMET ENTRY EOR X-DIP CURP'S
Ç
                                                                                 7386C
      IF(NX.NE.IPFEAS(NPRET)) CMATY(I,J-NPREIM)=(CC-CE)#FDG2
                                                                                 03870
```

•		SURVAT FRITEY FOR Y-DIR CHRISTS	03380
C		NOTE THAT EVEN QUE COST LEGATIVE	13890
•		ELB A-Die URchië	03900
350	CONTINUE		03910
_			03920
_	END JE WOMENT WAIRLY INTERVETION	P CONSTRUCTION	03930
C			93940
•	BESIN POUTINE TO ENTER HOMOGENER	US SOLIN EXPANSION COLIS IN MATRIX	03950
٢			03960
360	NBES=2*NPPE		03970
		HEGHEST ORDER BESSEL FUNCTION IN	03980
<b>r</b>		HUMCHENEUNS SUFIN EXBYMSION	03990
	IF (NINDX.=Q.2) NBFS =NRES -1		14000
۲		CHE LESS IF EVEN INDEX EXPANSION	04010
	D2 430 IM=1.4X		04020
	X=(FLPAT(IM)-0.500)+DX		04030
	07 400 JM=1.NY		04040
	Y=(FLAAT(JM)-0.500)*DY		24250
=	T = ( JM-1) * NX + I M		04060
•	DUT DATAMENTO A	INDEXING THOU MATCH-DTS	04070
	PHY=DATAN(Y/X)		04080
_	(Y*Y+X*X)**QP?(=PFR	20145 2000045 05 44754 075	04090
•		POLAR COORD'S OF MATCH-PTS	04100
	DPAFG=2*DIMAG(CS)*RHD		1411C
٢	DTMARG=-2*DREAL(GS)*RHO	ACCUMENT OF DECCH ENRE	04120
•	**************************************	APRUMENT OF BESSEL EN'S	04130
_	TELDABS(DIMARG/DRAPG).LT.1.E-27)		04140
Ç	CALL BECLIFFOR ADC DIMAGE DEBEC DI	TE FEAL APG SKIP TO FEAL BES CALL	04150
		MBES, MPES, O. DO. 16, TERF, DUM1, DUM2, D	2416C
_	1043+2044)	CET TABLE DE DESCEL EUROTTONE	24170
С	CD TO 3/0	GET TAPLE OF RESSEL FUNCTIONS	24180
364	- GO TO 368 - CALL RSLJZ(DRARG+DRRES+NRES+0+DO	14 TEED DIMS DIMS)	041 90 04200
304	CALL ZEROZ(DIMPES, 2*(NBES+1))	4 10 4 17 4 4 4 4 00 m 1 4 00 m 2 4	04210
	CALL ZEROZ(DIMRES, 2*(NRES+1))		04220
c	COLL 70405601 mule; 124644624111	SET UP PURE PEAL RES FUNCTIONS	04230
368	CC75T4(1)=0	3/ // // // // // // // // // // // // /	04240
,,,,	CSINTM(I)=0		142 50
<b>r</b>		ZERO IST TERM COFF CONSTRUCTION	04260
•		VELTURS	04270
	DO 370 IT=1.NPFEP1		04280
٢		INDEX THRU CALC OF CORE CONSTR	04290
C		VELTOR	24300
	XUNIN-11+2=X CMI		04310
^		CALC SERIES INDEX	04320
	*F(INDX.ED.D) GC TO 370		04330
5		SKIP CALC OF BELOW TERM FOR ZERO	04340
•		INDEX - IT WAS SET TO ZERO ABOVE	04350
	AR G=DFLDAT(INDX-1)*PHT		04360
C		ARGUMENT OF SIN EN	04370
	CRES=DCMPLX(DPRES(IMDX), DIMRES(I	NDX))	04380
	I 0 * 4 * 2 3 9 7 * 4 * 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		04390
	"SINTM(II)=DSIN(ARG) +CRES + 4+PI		04400
C		CALC COEFF CONSTRUCTION TERMS	04410
370	CONTINUE		04420
	nn 380 JJ=1.NPRFJ		04430
Ū		LOOP TO REPLACE COL'S FOR PREASSI	04440
Ċ		GMED J TERMS	04450
_	J= JPR F & S ( J J ) * M X		04460
_		INDEX OF COL BEING REPLACED	04470
	* NO X = ? * JJ - N * N O X	F3	04480
		51	

```
SERTES INCEX FOR PEPLACING TERM
C
                                                                                   04490
      G7 T7 (371,372), NSC CS
                                                                                   04500
ŗ
                                           SELECT PROPER SERIES OFFE ACCORDI
                                                                                   0451C
                                           NG TO Y SYMMETRY CONDITION
r
                                                                                   24522
                                           NSCES=> INDICATES COSINES IN X
                                                                                   04530
                                           CUEPENT ED
Ċ
                                                                                   04540
      CHOM(1,JJ)=-P1/(2*C5)*(0.D0,1.D0)**INDX*(CS[NTM(JJ)-CSINTM(JJ+1))
                                                                                   24550
      +(LL) MT2000) * XQNI **(00.1, C0.0) * (20* 9) \ IP == (LL, I+2H079N ) MC H0
                                                                                   94560
     1000ST4(JJ+11)
                                                                                   04570
      GO TO 380
                                                                                   04580
      -(+ttl)wf?nn)*('+XQAT)**(f,00.1.00)*(20*5)\[ -=(tt.f)MrH"
                                                                                   04590
     [(LL)MT? CON
                                                                                   14600
       +{LL}MT/127)*(!+Xd/11)**(!00.0)*(00.0)*(27*C)*(00.1)*(00.0)*(1)
                                                                                   74610
     ICSTNTM(JJ+U)
                                                                                  04620
     CONTINUE
                                                                                   ^463<sup>^</sup>
      DO 390 IT =1 ,NPREI
                                                                                  9464C
      J= { NY-1 } +NX+1 PPE&S(TT)+VPTCHS
                                                                                   04650
                                           LODD TO REPLACE COLIS FOR PREASS!
                                                                                   14660
      JJ=TT+NPREJ
                                                                                  04670
                                           GNED T TERMS
                                                                                  04680
      INDX=2*(II+NPFEJ)-NINDX
                                                                                  04690
      GT TO (381,382), MSC PS
                                                                                  04700
      CHOM( 1, JJ )=-P1/(2*f5)*(0.D0, 1.D0)*****DX*(CS[MTM(TI+NPREJ)-
                                                                                  04710
     1CSINTM([]+NPRFJ+1])
                                                                                  04720
      CHTM(I+MPTCHS,JJ) =-P!/(2*CS)*(0.D0,1.D0)**INDX*(CCCSTM(TI+MPFFJ)+
                                                                                  04720
     1CCOSTM(IT+NPRFJ+1))
                                                                                  24740
      30 TO 390
                                                                                  04750
 382 CH7M(I,UJ)=-PT/(2*C$)*(0.0.0,1,+00)**(INDX+1)*(CCOSTM(TT++PRFU+1)-
                                                                                  74760
     1000574(II+NPPEJ))
                                                                                  24770
      CHTM([+NPTCHS,JJ) =-PT/(2 *C c) * (0 . DO , 1 . DO) ** (INDX+1) *
                                                                                  94780
     1(CSIVIM([T+VPREJ]+CCTNTM(IT+NPREJ+1))
                                                                                  04790
 390
     CONTINUE
                                                                                  24801
      CONTINUE
 400
                                                                                  04810
^
                                                                                  04820
      END DE MOMENT MATRIX CONSTRUCTION
                                                                                  24836
                                                                                  04940
      CONTINUE
 405
                                                                                  04850
      COLL SPRHOM(CMATX, NOTCHS, NOTCHS-MPREJ, MOIMI, MOIMI,
                                                                                  24860
                    CMATY, NOTCHS, NOTCHS-NOFET, NOINT, NOINT,
                                                                                  04370
     1
                   CHOM. NOTWOI. NOIMOJ, CM&T4, NOIMOJ, NOIMI, CDET)
     2
                                                                                  04880
      FPAT=CDABS(CMATX(1,1))
                                                                                  24890
      CPLATE=CDET/FRAT
                                                                                  04900
      WRITE(6, 20) CSUNDR, CPLATE
                                                                                  04910
20
      FORMAT(* *,5X,2F12.4,5X,2F12.4)
                                                                                  04920
      RETURN
                                                                                  04930
      END
                                                                                  14940
```

```
SUBFOUTINE SPRHOM (CMAT1, NI1, NJ1, NDIM11, NDIM1J, CMAT2, NI2, NJ2, NDIM2I
                                                                                  00010
      1, NDI M2J, CMAT3, NDI M3I, NDI M3J, CMAT4, NDI M4I, NDI M4J, CDET)
                                                                                  00020
       IMPLICIT COMPLEX*16(C), REAL*8(A,B,D-H,C-Z)
                                                                                  00030
C
                                                                                  00040
C
       SUBROUTINE TO DIAGONALIZE AND CALC DETERMINANT OF A SPARCELY-
                                                                                  00050
C
       COUPLED MATRIX
                                                                                  00060
C
       BY L W PEARSON 7/74
                                                                                  00070
      REVISED 5/75
                                                                                  00075
C
                                                                                  00080
       DIMENSIGN CMAT1(NCIM11,NDIM1J), CMAT2(NDIM21,NDIM2J), CMAT3(NDIM3I,
                                                                                  00090
      INDIM3J).CMAT4(NDIM4I.NDIM4J)
                                                                                  00100
       NI3=NI1+NI2
                                                                                  00110
       NJ3=NI3-NJ2-NJ1
                                                                                  00120
       CALL ZEROZ(CMAT4, 4*NDIM4I*NDIM4J)
                                                                                  00130
       CDET =1
                                                                                  0 CL 40
                                          INITIALIZE PRODUCT ACCUMULATOR
C
                                                                                  CC150
       NPR=3
       NJ1M1=NJ1-1
                                                                                  C0160
       NJ1L=NJ1
       IF(NJ2+NJ3.GE.1) GO TO 95
       NJ1L=NJ1L-1
      NPR = 1
 95
       DO 155 M=1.NJIL
                                                                                  00170
                                          INDEX ACROSS COL
                                                                                  001 80
       MP 1=M+1
                                                                                  00190
       FMAX=CDABS(CMAT1(M,M))
                                                                                  C C2 CO
      K=M
                                                                                  00210
      IF(MP1.GT.NI1) GO TO 1G5
                                                                                  00220
      DO 100 I=MP1,NI1
                                                                                  00230
                                          LGCF TO SEARCH FOR PIVOT IN MTH
C
                                                                                  00240
C
                                          COL
                                                                                  00250
      FCK=CDABS(CMATI(I,M))
                                                                                  00260
      IF(FCK.LE.FMAX) GC 70 100
                                                                                  00270
      K= I
                                                                                  00280
                                          IF LARGER ELEMENT FOUND MARK ROW
C
                                                                                  00290
      FMA X=FCK
                                                                                  00300
C
                                          USE NEW LARGE ELEMENT AS COMPARI-
                                                                                  00310
                                          SON VALUE
                                                                                  C0320
C
 100
      CONTINUE
                                                                                  00330
      CSTOR=CMAT1(K,M)
 105
                                                                                  00340
C
                                          SAVE VAL OF PIVOT ELEMENT
                                                                                  00350
                                                                                  00360
      CDET=CDET+CSTOR
C
                                          MULT PIVOT INTO PROD ACCUMULATOR
                                                                                  CC370
      IF (K.EQ.M) GO TC 115
                                                                                  00380
C
                                          IF PIVOT CN DIAG SKIP RGW EXCH
                                                                                  00390
      CDET =-CDET
                                                                                  00400
C
                                          CHANGE SIGN BECAUSE OF ROW EXCH
                                                                                  00410
 107
      DD 110 J=M,NJ1
                                                                                  00420
C
                                          LOOP TO EXCH DIAG AND PIVOT RUWS
                                                                                  00430
      CSTC=CMAT1(K,J)
                                                                                  00440
      CMAT1(K, J)=CMAT1(M,J)
                                                                                  00450
      CMAT1(M, J) = CSTC
                                                                                  00460
 110 CONTINUE
                                                                                  00470
      IF(NJ3.LT.1) GO TO 115
                                                                                  00475
      DO 112 J=1,NJ3
                                                                                  C0480
      CSTO=CMAT3(K,J)
                                                                                 00490
      CMAT3(K,J)=CMAT3(M,J)
                                                                                  CC5CO
      CMAT3(M,J)=CSTO
                                                                                  00510
      CONTINUE
                                                                                  00520
 115 CONTINUE
                                                                                 00560
      IF (MP1.GT.NI1) GO TO 155
                                                                                 00570
```

	ACT TO SECURE A PROPERTY OF THE PARTY OF THE		
365,000	00 150 I=MP1.NI1		00580
C	CT. C CH. T. I C N. ICCO.	ELIMINATION LUCE FOR CHATI	00590
	CFAC=CMAT1(I, H)/CSTOR	PI THE ATTON PACTOR	00600
C	IF(MP1.GT.NJ1) GO TO 125	ELIMINATION FACTOR	00610
	DO 120 J=MP1, NJ1	THE REPORT OF THE PERSON OF TH	60630
C	00 120 3-FF14N31	LOOP ACROSS ROW IN CMATS	00640
	CMAT1(1.J)=CMAT1(1.J)-CMAT1(M		00650
120	CONTINUE	703.00	00660
	IF(NJ3.LT.1) GO TC 150		00665
125	00 130 J=1,NJ3		00670
C		LOGP ACROSS ROW IN CMAT3	00680
	CMAT3( I, J) = CMAT3(I, J) - CMAT3( M	,J)*CFAC	00690
130	CONT INUE		00700
150	CONTINUE		00720
155	CONTINUE		00730
	NI4=NI 1-NJ1		00740
	IF(NI4.LE.O) GO TC 290		00750
C			00760
C	BEGIN ROUTINE TO CREATE/ DIAG	ONALTZE CMATA	00770
č	BEGIN ROUTINE TO CREATE VIDING	UNALIZE CHAIA	00790
770	NPI V=NI 4		00800
	IF(NI4.GT.NJ2) NPIV=NJ2		00810
	DO 250 M=1, NPIV		00820
C		INDEX ACROSS COL FGR CMAT4 CIAG	00830
	MP 1=M+1		00840
	FMAX=CCABS(CMAT2(1,M))		00850
	K=1		00860
	IF(N12-LT-2) GO TO 205		00870
19.8	DO 200 I=2,NI2	1000 TO CEADOL COD DAVIOR AN WELL	08800
Ç		LOCP TO SEARCH FOR PIVOT IN MTH	00890
C	FCK=CDABS(CMAT2(1,F))	CUL	00900
	IF(FCK.LE.FMAX) GC TC 200		00920
	K=1		00920
C		IF LARGER ELEMENT FOUND MARK ROW	00940
	FMAX=FCK		00950
C		USE NEW LARGE ELEMENT AS COMPARI-	00960
C		SON VALUE	00970
200	CONTINUE		00980
205	CSTOR=CMAT2(K,M)		C0990
C	ADPT - ADPT - ADDT - AD	SAVE VAL CF PIVOT ELEMENT	01000
18	CDET=CDET+CSTOR	MILE BEING THE SECOND	01010
C	CDET=-CDET	MULT PIVOT INTO PROD ACCUM	01020
c	CUE (==CUE I	CHANGE SIGN OF DETERM BECAUSE OF	01030 01040
C		EXCHANGE FROM CMAT2 TO CMAT4	01050
	DD 210 J=M.NJ2	ENGINEE FROM COME TO COME	01060
С	20 ERG 6-114110E	LOOP TO EXCHANGE DIAG AND PIVOT R	01070
Č		ROWS	01080
	CSTO=CMAT4(M,J)		01090
	CMAT4(M,J)=CMAT2(K,J)		01100
	CMAT2(K, J)=CSTC	,	01110
210	CONTINUE		01120
	K3=K+NI1		01130
	M3=NJ1+M		01140
	IF(NJ3.LT.1) GO TO 213		01145 01150
	DO 212 J=1,NJ3 CSTO=CMAT3(K3,J)		01160
	CMAT3(K3,J)=CMAT3(H3,J)		C1170
	Onni Jinjyu/uraijinjyu/	54	21110

			11.725
	CHAT3(M3,J)=CSTO		01180
212	CONT INUE		01190
213	IF(NI2.LT.1) GC TC 290		01225
235	DO 250 I=1.NI2		01230
	DU 230 1-14412	LOOP TO CARRY ELIMINATION INTO	
C			01240
C		CMAT2	01250
	I3=NI1+I		01260
	CFAC=CMAT2(I,M)/CSTOR		C1270
	IF(MP1.GT.NJ2) GO TC 242		01280
_	DD 240 J=MP1,NJ2		01290
C		LOCP ACROSS ROW OF CMAT2	01300
	CMAT2(I,J)=CMAT2(I,J)-CMAT4(P,J)	) *CFAC	01310
240	CONTINUE		C1320
240	IF(NJ3.LT.1) GO TO 250		
			01325
242	DO 245 J=1,NJ3		C1330
C		LOOP ACROSS ROW OF CMATS	01340
	CMAT3(13,J)=CMAT3(13,J)-CMAT3(N.	3.J)*CFAC	01350
245	CONTINUE		01360
250	CONTINUE		01380
C			01390
С	END ROUTINE TO 'CIAGONALIZE' CM	AT 4	G1400
C			01410
290	IF(N14.GE.NJ2) GO TO 350		01420
	11 (414-66-4027 00 10 370	TE STACOUAL DOCK WAT DARK THOU	
C		IF CIAGONAL DOES NOT PASS THRU	C1430
C		SKIP CIAGONALIZATION FOR CMAT2	01440
C			01450
C	BEGIN ROUTINE TO 'CIAGONALIZE'	CMAT 2	C1460
Č			
C	NT ( D 1 = N 2 4 4 1		01470
	NI4P1=NI4+1		01480
	NJ2L=NJ2		01482
	IF(NJ3.GE.1) GC TC 295		01484
	NJ2L=NJ2L-1		01486
	NPR=2		01488
295	DO 350 M=NI4P1, NJ2L		01492
	MI=M-NI4		C1500
	M3=MI+NI1		C1510
	MP 1=M+1	,	01515
	MIP1=MI+1		01520
	FMAX=CDABS(CMAT2(FI,F))		C1530
	K=MI		01540
	IF(MIP1.GT.NI2) GO TO 305		C1550
	DO 300 T=MIP1,NI2		01560
_	OO DOO LEMENTANIE	1000 TO CEARCH FOR STACE TA HT	
C		LOCP TO SEARCH FOR PIVET IN MTH	01570
C		COL	01580
	FCK=CDABS(CMAT2(I,F))		01590
	IF(FCK.LE.FMAX) GO TO 300		01600
	K=T		01610
	N=4	TE LARGER FLENCHS COULD HARD SOL	01010
C			01155
		IF LARGER ELEMENT FOUND MARK ROW	01620
	FMAX=FCK	IF LANGER ELEMENT FOUND MARK NOW	01620 C1630
С	FM AX = FCK	USE NEW LAPGE ELEMENT AS COMPARI-	C1630
C	FM AX = FCK	USE NEW LAPGE ELEMENT AS COMPARI-	C1630 01640
C			01640 01650
C 300	CONTINUE	USE NEW LAPGE ELEMENT AS COMPARI-	C1630 01640 01650 01660
300 305		USE NEW LARGE ELEMENT AS COMPARI- SON VALUE	C1630 01640 01650 01660 01670
C 300	CONTINUE	USE NEW LAPGE ELEMENT AS COMPARI-	C1630 01640 01650 01660
300 305	CONTINUE CSTOR=CMAT2(K,M)	USE NEW LARGE ELEMENT AS COMPARI- SON VALUE	C1630 01640 01650 01660 01670 01680
300 305	CONTINUE CSTOR=CMAT2(K,M) K3=K+NII	USE NEW LARGE ELEMENT AS COMPARI- SON VALUE	C1630 O1640 O1650 O1660 C1670 O1680 O1690
300 305 C	CONTINUE CSTOR=CMAT2(K,M)	USE NEW LAPGE ELEMENT AS COMPARI- SON VALUE  SAVE VAL CF PIVOT ELEMENT	C1630 O1640 O1650 O1660 O1670 O1680 O1690 O1700
300 305	CONTINUE CSTOR=CMAT2(K,M) K3=K+NI1 CDET=CDET*CSTOR	USE NEW LARGE ELEMENT AS COMPARI- SON VALUE	C1630 01640 01650 01660 01670 01680 01690 01700
C 300 305 C	CONTINUE CSTOR=CMAT2(K,M) K3=K+NII	USE NEW LARGE ELEMENT AS COMPARISON VALUE  SAVE VAL CF PIVOT ELEMENT  MULT FIVOT INTO PROD ACCUMULATOR	C1630 01640 01650 01660 01670 01680 01690 01700 01710
300 305 C	CONTINUE CSTOR=CMAT2(K,M) K3=K+NI1 CDET=CDET*CSTOR	USE NEW LAPGE ELEMENT AS COMPARI- SON VALUE  SAVE VAL CF PIVOT ELEMENT	C1630 01640 01650 01660 01670 01680 01690 01700
C 300 305 C	CONTINUE CSTOR=CMAT2(K,M)  K3=K+NI1 CDET=CDET+CSTOR  IF(K.EQ.MI) GO TC 315	USE NEW LARGE ELEMENT AS COMPARISON VALUE  SAVE VAL CF PIVOT ELEMENT  MULT FIVOT INTO PROD ACCUMULATOR	C1630 01640 01650 01660 01670 01680 01690 01700 01710 01720 01730
C 300 305 C	CONTINUE CSTOR=CMAT2(K,M) K3=K+NI1 CDET=CDET*CSTOR	USE NEW LARGE ELEMENT AS COMPARISON VALUE  SAVE VAL CF PIVOT ELEMENT  MULT FIVOT INTO PROD ACCUMULATOR	C1630 01640 01650 01660 01670 01680 01690 01700 01710

	DO 310 J=M, NJ2		C1 760
	CSTO=CMAT2(K,J)		01770
	CMAT2(K,J)=CMAT2(MI,J)		01780
	CMAT2(MI.J)=CSTC		01790
310	CONTINUE		01800
	IF(NJ3.LT.1) GO TC 315		C1805
	DO 312 J=1,NJ3		01810
C		LOOP FOR EXCH IN CMAT3	01820
	CSTO=CMAT3(K3.J)		01830
	CMAT3(K3,J)=CMAT3(M3,J)		01840
	CMAT3(M3.J)=CSTO		C1850
312	CONTINUE		01860
315	CONTINUE		01900
	IF(MIP1.GT.NI2) GO TO 390		C1910
	DO 350 I=MIP1,NI2		01920
C		ELIPINATION LOOP	01930
•	I3=I+NI1	CELLINATION COOL	C1940
	CFAC=CMAT2(I,M)/CSTOR		01950
	IF(MP1.GT.NJ2) GO TO 335		01960
	DO 330 J=MP1.NJ2		C1970
С	DU 330 3-6-14M32	LCCF ACROSS ROW OF CMAT2	01980
•	CM AT 2( I, J) = C MAT 2( I, J) - CM AT 2(MI, J		01990
330	CONTINUE	7+CFAC	02000
335	IF(NJ3.LT.1) GO TC 350		02005
333	DO 345 J=1.NJ3		
C	00 343 J-1,NJ3	LOOP ACROSS ROW IN CMAT3	C2010
C	CMATALIA 11-CMATALIA 11-CMATALMA		02020
245	CMAT3(13, J)=CMAT3(13, J)-CMAT3(M3 CONTINUE	,J/+CFAC	02030
345 350	CONTINUE		02040
	CONTINGE		C2060
C C	DECTN DOUTTNE TO ACTACONAL TICE C	MATS	02070
	BEGIN ROUTINE TO 'CIAGONALIZE' C	MAIS	C2080
C	N12M1-N12 4		02090
390	NJ3M1=NJ3-1		02100
	IF(NJ3M1-LT-1) GC TC 455		02110
	DO 450 M=1.NJ3F1	****** *******************************	02120
C		INDEX ACROSS COL	02130
	MP1 = M+ 1		02140
	MI=M+NJ1+NJ2		02150
	MIP1=MI+1		C2160
	FMAX=CDABS(CMAT3(MI,M))		02170
	K=MI		02180
	IF(MIPL.GT.NI3) GC TO 405		C21 90
_	DO 400 I=MIP1,NI3		02200
Ç		LOCP TO SEARCH FOR PIVOT IN MTH	02210
C		COL	C2220
	FCK=CDABS(CMAT3(I,M))		02230
	IF(FCK .LE.FMAX) GO TO 400		02240
	K=I		02250
C		IF LARGER ELEMENT FOUND MARK ROW	02260
	FM AX = FCK		C2270
С		USE NEW LARGE ELEMENT AS COMPARI-	02280
С		SO VALUE	02290
400	CONTINUE		C2300
405	CSTOR=CMAT3(K,M)	40 - 000	02310
C		SAVE VAL CF PIVOT ELEPENT	02320
	CDET = CDET + CST OR		C2330
С		MULT FIVOT INTO PROC ACCUMULATOR	02340
	IF(K.EC.MI) GC TO 415		C2350
C ,		IF PIVOT ON DIAG SKIP ROW EXCH	02360
	CDET=-CDET		02370
C		CHANGE SIGN BECAUSE OF ROW EXCH	C2380
	56		

	DO 410 J=M.NJ3		02390
C		LOOP TO EXCH DIAG AND PIVOT ROWS	02400
	CSTO=CMAT3(K,J)		02410
	CMAT3(K,J)=CMAT3(MI,J)		02420
	CMAT3(MI,J)=CSTO		C2430
410	CONTINUE		02440
415	CONT INUE		02480
	DO 450 I=MIP1, NI3		02490
C	The state of the s	ELIMINATION LOOP	02500
	CFAC=CHAT3(I.M)/CSTOR		02510
	DO 445 J=MP1,NJ3		02520
C		LOOP ACROSS RCW IN CMATS	02530
200	CMATS! I.J) = CMATS(I.J) - CMATS(MI.J	)+CFAC	02540
445	CONTINUE		02550
450	CONT INUE		02570
455	GO TO 1461,462,4631, NFR		02572
461	CDET=CDET+CMAT1(NI1,NJ1)		02574
	RETURN	A.	C2576
462	CDET=CDET+CMAT2(NI2.NJ2)		02578
	RETURN		02582
463	CDET=CDET+CMAT3(NI3.NJ3)		02584
	RETURN		02600
C	1.00	MULT LAST ELEMENT INTO DETERM	02590
-	END	THE THE PERSON AND SELECTION	02610

The second	SUBROUTINE SPRSLV (CMAT 1, NI 1, NJ 1, 1, ND 1,	NDIM11, NDIM1J, CMAT2, NI2, NJ2, NDIM2I [4, NDIM41, NDIM4J, CRHS, CSCLN]	00010 00020
CC	SUBROUTINE TO BACKSCLVE A TRIANG	GULARIZED SYSTEM OF SPARCELY-	00030 00040 00050
CCC	BY L W PEARSON 7/74 REVISED 5/75		00052 00054 00056
C	STORAGE FORM COMPATIBLE WITH THE	TRIANGULARIZATION ROUTINE SPARCE	00060 00070
C C C		THE SOLUTION FOR NATURAL VECTORS THE DETERMINANT OF THE SYSTEM IS	00080 00090 (\0100 00110
	IMPLICIT COMPLEX*16(C), REAL*8(A; DIMENSION CMATI(NDIMII, NDIMIJ), C IDIM3J), CMAT4(NDIM4I, NDIM4J), CRHS LOGICAL LHOM	CMAT2(NDIM21, NDIM2J), CMAT3(NDIM31, N	00120 00130 00140 00150
CCC	SETUP FOR INHOPOGENEOUS SYSTEM		00160 00170 00180
	LHCM=.FALSE.		00190
C	NI 3=NI 1+NI2	SET INDICATOR FOR INHOM ENTRY	00200 00210
C	M 4 3 - N 7 3 - A 4 4 3 - A 4 4 3	NO ROWS IN COUPLING SUBMATRIX	00220
С	NJ 3=NI 3-NJ1-NJ2	NO CF COLS	00230 00240
	NI4=NI1-NJ1		00250
C		NO POWS IN SECONDARY COUPLING SUBMATRIX	00260 00270
_	ND2=NJ2-NI4	NO OF DIAGONAL PROME OF MARROW	00280
C		NO OF DIAGGNAL TERMS OF MATRIX IN CMAT2	00290 00300
	NPR=3 IF(NJ3.LT.1) NPR=2		
C C	IT THO SECTION THE NAME OF THE SECTION OF THE SECTI	SET INDICATOR FOR NULL CMAT3 DEGENERACY	
	IF(NJ3+NJ2.LT.1) NPR=1		
C C		SET INDICATOR FOR NULL CMAT2 & CMAT3	
С	GO TO (81,82,83) , NPR	CO MAKE STORT DIVISION SOR DICHT	
Č		GO MAKE FIRST DIVISION FOR RIGHT-	
81	CSCLN(NI3)=CRHS(NI3)/CMATI(NII,N GO TO 10C		
82	CSOLM(NI3)=CRHS(NI3)/CMAT2(NI2, N GO TO 100	J2)	
83	CSOLN(NI 3) = CRHS(NI3)/CFAT3(NI3,N		
C	GO TO 100	SOLVE FOR "LAST" UNKNOWN	00320 00330
C C	GB 10 100	GO TO SCLN FOUTINES	00340 00350
C	END OF SETUP FOR INHEM SYSTEM		00360 00370
C	BEGIN ENTRY/SETUP FOR HOMOGENEOU	S SYSTEM	00380 C0390
С	ENTRY HOMSLV(CMAT1.NI1.NJ1.NDIM1 12J,CMAT3.NDIM3I.NCIP3J,CMAT4.NDI	I, NDIMIJ, CMAT2, NI2, NJ2, NDIM2I, NDIM M4I, NCIM4J, CSOLN, NORD)	00400 00410 00420
C	LHCF=.TRUE.		0C430
C	1000	LOGICAL INDICATOR FOR HOMOGEN SYS	00440

	NI3=NI1+NI2		00450
	NJ3=NI3-NJ1-NJ2		00460
	NI 4=NJ 1-NJ 1		00470
	ND2=NJ2-NI4		00480
	CSGLN(NI3)=1		00490
С	CSUEMINIST -1	ASSIGN ARBITRARY ELEMENT IN SCL'N	00500
č		ASSIGN ANDITHAN CEEDEN IN SEC 1	CC510
Č	END SETUP FOR HOPEGENEOUS ENTRY		00520
C	END SETUP FOR HUPLGIPECOS ENTRY		00530
Č	BECTH BACKEDINE FOR FOUNTTONE IN	VOLVING ONLY CHATS (LAST NJS EQS)	00550
	DEGIN DUCKAULAE LOL EMONITONS IN	ANTATUR MUEL CHAID LEADI MID EADI	
C			00550
100	FMAX=CCABS(CSOLN(NI3))		00560
	IMAX=NI3		00570
-	IF(NJ3.LT.2) GC TC 2CO	**** BOUTENE ** OLIV **** WARENE	00580
C		SKIP ROUTINE IF ONLY LAST VARIABL	00590
C		COUPLES (IT WAS SOLVED/ASSIGNED	00600
r,		ABOVE)	00610
	DO 150 IC=2,NJ3		CC620
	ICM1=IC-1		00630
	I=NI3-· IC+1		00640
	I=NI3-IC+1		00/250
C		CALC MATRIX ROW INCX FROM	20660
C		COMPLEMENTARY INDX	00670
•	JD3=I~AJ1-NJ2		C0680
C		COL INDX FCR CMAT3 WHICH DEFINES	00690
č		DIAG CF MATRIX	00700
·	CSUM=0	DING CI MAINIA	00710
			00720
•	00 110 J3C=1,ICM1	LOOP TO ACCUM NEGATIVE SUM CF	00720
C			
С		PREVIOUSLY CALC'D UNKNS	00740
_	J3=NJ3+1-J3C		00750
C	. 102 - 11 - 22	COL OF COEF IN CMAT3	00760
	J=NI 3+1-J3C		00770
C		ROW OF UNKN IN CSCLN	09780
	CSUM=CSUP-CMAT3(I,J3)*CSOLN(J)		00790
110	CONTINUE		00800
	IF(.NOT.LHOM) CSUM=CSUM+CRHS(I)		00810
C		ADD R H S TO SUM	00820
	CSOLN(I)=CSUM/CFAT3(I,JD3)		00830
C		DIVIDE BY DIAG COEF	00840
_	IF (CDABS (CSOLN(I)).LE.FMAX) GO T	0 150	CO8:50
	FMAX=CDABS(CSCLN(I))		00860
	IMAX=I		00870
C	IMAX-1	CHECK FOR MAX ELEMENT	00880
-	CONTINUE	CHECK FOR HAN EELHEN	00890
150	CONTINUE		00890
C		ATC THUCKUTAC CHATS C CHATS	
C	BEGIN ROUTINE TO SCLVE FOR ELEME	NIS INVULVING CHAIS & CHAIZ	00910
C			00920
200	IF(NJ3.GE.NI2) GC TO 300	4440 044E45 55 0540 0056 40E	00930
C		SKIP ROUTINE IF DIAG DOES NOT	00940
C		PASS THRU CMAT2	00950
	DO 250 IC=1.NC2		CC960
	ICM1=IC-1		00970
	12=N12-NJ3+1-1C		00980
	I3=NI3-NJ3+1-IC		00990
	JD 2=NJ 2+ 1-IC		01000
	NCM1=NJ3+IC-1		01010
	CSUM=0		01020
	IF(NJ3.LT.1) GO TO 215		
	DO 210 JC=1,NJ3		01030
c	00 210 907171103	LOOP TO SUM CONTRIB FROM CHAT3	01040
10TH	59	COU. TO DET CONTINGO FROM CIRTS	

	"J3=NJ3+1-JC	01050
	J=NI3+1-JC	01060
	CSUM=CSUM-CHAT3(I3,J3)*CSCLN(J)	01070
210	CONTINUE	01080
215	IF(ICMI-LT.1) GO TO 225	01090
C	SKIP IF NO TERMS CONTRIB FR CMAT2	61100
	DO 220 J2C=1,1CM1	01110
	J2=NJ2+1-J2C	01120
	J=NI3-NJ3+1-J2C	61130
	CSUM=CSUM-CMAT2(12.J2) +CSOLN(J)	01140
220	CONTINUE	01150
225	IF(.NOT.LHOM) CSUM=CSUM+CRHS(I3)	C1160
263	CSOLN(13)=CSUM/CMAT2(12, JD2)	01170
	IF(CDABS(CSOLN(I3)).LE.FMAX) GO TO 250	
	FMAX=CDABS(CSCLN(13))	01180 01190
250	IMAX=I	01200
250	CONTINUE	01210
C		01220
C	BEGIN ROUTINE TO SOLVE FOR ELEMENTS INVOLVING CMAT3 6CMAT4	01230
C		01240
300	IF(NI4.LT.1) GC TC 400	
	DO 350 IC=1,NI4	
	14=NI4+1-IC	01260
	JD4~14	01270
	I3=NI1+1-1C	01280
	CSUM=0	01290
	IF(NJ3.LT.1) GO TO 315	
	DO 310 J3C=1,NJ3	C1300
	J3=NJ3+1-J3C	01310
	J=NI3+1-J3C	C1320
	CSUM=CSUM-CMAT3(I3,J3) +CSGLN(J)	01330
310	CONT INUE	01340
315	NSUBS=NC2+IC-1	01350
C	NG CF NON-DIAG CMAT4 EL S IN EQ	01360
•	IF(NSUBS.LT-1) GO TO 325	01370
	DO 320 J4C=1 + SUBS	01380
	J4=NJ2+1-J4C	01390
	J=N13-NJ3+1-J4C	C1400
	CSUM=CSUM-CMAT4(I4,J4)*CSOLN(J)	61410
320	CONTINUE	01420
325	IF(.NOT.LHCM) CSUM=CSUM+CRHS(13)	C1430
363	CSOLN(I3) = CSUM/CMAT4(I4, I4)	01440
	IF(CDARS(CSOLN(I3)).LE.FMAX) GO TO 350	
	FMAX=CDABS(CSCLN(13))	C1450 C1460
		_
350	I MA X=I 3 CONT INUE	01470
	CONTINUE	01480
Č	DEGIN POLITICE TO SELLE FOLK TANGLUTING CHATA & CHATA	C1490
C	BEGIN ROUTINE TO SOLVE EC'S INVOLVING CMATS & CMATS	C15C0
C		01510
400	IF(NJ1-LT-1) GC TC 455	
	DO 450 IC=1,NJ1	01520
	I=NJ 1+1-IC	01530
	ICM1=IC-1	C1540
	CSUM=0	01550
	IANJ3-LT-11 GO TO 415	
	DO 410 J3C=1, AJ3	C1560
	J3=NJ3+1-J3C	01570
	J=N13+1-J3C	01580
	CSUM=CSUM-CMAT3(I,J3)+CSCLN(J)	01590
410	CONTINUE	01600
415	IF(ICHI.LT.1) GC TC 425	01610
	60	

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- 10

	DO 420 JC=1, ICM1	©1620
	J=NJ1+1-JC	01630
	CSUM=CSUM-CMAT1(I.J) +CSOLN(J)	01640
420	CONTINUE	01650
425		
423	IF(.NOT.LHOM) CSUM=CSUM+CRHS(I) CSOLN(I)=CSUM/GMATI(I.I)	01660
		01670
	IF(CDABS(CSOLN(I)).LE.FMAX) GO TO 450	01680
	FMAX=CDABS(CSCLN(I))	01690
	IMAX=I	01700
450	CONTINUE	01710
C		01720
C	The second second second	01730
C	END OF SOLUTION	01740
C		01750
455	IF(.NOT.LHOM) RETURN	01760
C	RETURN IF INHOM SYSTEM	01770
C		01780
C	BEGIN NORMALIZATION ROUTINE FOR NATURAL VECTOR FOR HOMOGENEOUS	01790
C	CASE	01800
С		01810
	CSCALE=1./CSOLN(IMAX)	01820
	DO 500 I=1,NI3	C1 83 0
	CSGLN(I) = CSGLN(I) + CSGALE	C1840
500	CONTINUE	01850
	RETURN	C1860
	END	01870

SUBROUTINE COPYZ(X,Y,N) DIMENSION X(1),Y(1) DO 100 I=1,N X(I)=Y(I) 100 CONTINUE RETURN END	09340 09350 09360 09370 09380 09390

	SUBPOUTINE ZEROZ(IARRAY.N)	
	DIMENSION TARRAY(1)	99410
	DO 100 T=1.N	09420
	TARRAY(1)=0	09430
100	CONTINUE	09440
	RETURN	09450
	END	09460
		09470

	SUBROUTINE DWEDDL(FCN, N, DELTA, VINT)	09480
	IMPLICIT REAL+8(A-H,0-Z)	09490
	COMPLEX*16 FCN.C.VINT	0.95.00
	DIMENSTON FON(N)	09510
	DIMENSION COEF(6)	09520
	DATA CREF/2.00.5.00.1.00.6.00.1.00.5.00/	09530
	[F((N-1)/6+6.EQ.N-1) GO TO 100	09540
	WRITE (6,1)	09550
1	FORMAT( OINCORRECT POINTS TO WEDDLE!)	^9560
7	A=1/0	09570
100	CONTINUE	09580
	VINT=0	19590
	DO 200 J=1,N	09600
	JC2FF=J-((J-1)/6)*6	19610
	VINT=VINT+COEF(JCOEF)*FCN(J)	09620
200	CONTINUE	09630
	VINT=(VINT-FCN(1)-FCN(N))*(0.3D0,0.D0)*DCMPLX(DELTA,0.D0)	09640
	RETURN	09650
	END	09660

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.ZAN0967U
                                                                             ZAN09580
C
                          - DETERMINATION OF ZEROS OF AN ANALYTIC COMPLEX ZANO9690
    FUNCTION
                              FUNCTION USING MULLER'S METHOD WITH
C
                                                                             ZAN09700
C
                              DEFLATION
                                                                             ZAN09710
    USASE
                          - CALL ZANLYT (F. EPS. NSIG. KN. NGUESS, N. X. ITMAX,
                                                                             Z4N09720
                              INFEP . TER)
                                                                             ZAN09730
C
    PARAMETERS
                  F
                          - A FUNCTION SUBPROGRAM, F(Z), WRITTEN BY THE
                                                                             ZAN09740
                              USER SPECIFYING THE EQUATION WHOSE ROOTS ARE TO BE FOUND. F MUST BE TYPE-NAMED AS
C
                                                                             ZAN09750
C
                                                                             ZAN09760
                              FOLLOWS - COMPLEX FUNCTION F*16 (Z)
                                                                             Z4N09770
C
C
                          - 1ST STOPPING CRITERION. A ROOT Z IS ACCEPTEDZANO9780
                  EPS
C
                              IF ABSOLUTE VALUE OF F(Z) .LE. EPS (INPUT)
                                                                             Z4N09790
                          - 2ND STOPPING CRITERION. A ROCT IS ACCEPTED
                                                                             Z4N09800
                  NSIG
                              IF TWO SUCCESSIVE APPROXIMATIONS TO A GIVEN ZAMO9810
C
C
                              ROOT AGREE IN THE FIRST NSIG DIGITS. (INPUT) ZANO9820
                                NOTE. IF EITHER OR BOTH OF THE STOPPING
¢
                                                                             ZAN09830
                                CRITERIA ARE FULFILLED, THE ROOT IS
¢
                                                                             ZAN09840
C
                                ACCEPTED.
                                                                             ZAN09850
                          - THE NUMBER OF KNOWN ROOTS WHICH MUST BE STOREDZANO9860
Ç
                  ΚN
                              TV X(1),...,X(KN), PRIOR TO ENTRY TO ZANLYT ZANO9870
Ç
C
                  NGUESS - THE NUMBER OF INITIAL GUESSES PROVIDED. THESE ZANO9880
C
                              GUESSES MUST BE STORED IN X(KN+1).....
                                                                             ZAN09890
C
                              X(KN+NGHESS) AND NGUESS MUST BE SET EQUAL
                                                                             Z4N09900
C
                              TO ZERO IF NO GUESSES ARE PROVIDED. (IMPUT) ZANO9910
C
                          - THE NUMBER OF NEW POOTS TO BE FOUND BY
                                                                             ZAN09920
C
                              ZANLYT (INPUT)
                                                                             ZAN09930
r,
                          - A LONG-WORD COMPLEX VECTOR ARRAY OF LENGTH
                                                                             ZAN09940
C
                              .GE. 3+(KN+N). X(1),...,X(KN) ON INPUT
                                                                             ZAN09950
                              MUST CONTAIN ANY KNOWN POOTS. X(KN+1),...,
                                                                             ZAN09960
C
C
                              X(KN+N) ON INPUT MAY, AT THE USER'S OPTION, ZANO9970
                              CONTAIN INITIAL GUESSES FOR THE N NEW
                                                                             ZAN09980
C
                              POOTS WHICH ARE TO BE COMPUTED. ON CUTPUT,
                                                                             ZAN09990
                              X(KN+1),..., X(KN+N) CONTAIN EITHER A ROOT
C
                                                                             ZAN10000
CORRECT TO WITHIN A CONVERGENCE CRITERON
                                                                             ZAN10010
Ç
                              OR THE VALUE(12345678.123456780+0,12345678.
                                                                            ZAN10020
C
                              12345678D+0) INDICATIVE OF A FAILURE TO
                                                                             ZAN10030
                                                                            ZAN10040
C.
                              ACHIEVE THE SPECIFIED CONVERGENCE FOR THAT
c
                              POOT , SAY X(KN+J). IN THE LATTER CASE, THE
                                                                            ZAN10050
                              MOST RECENT APPROXIMATION TO X(KN+J) IS
                                                                             ZAN10060
Ç
                              AVATLABLE IN X(ISUB). WHERE ISUB=2*(KN+N)+J ZAN10070
C
                         - THE MAXIMUM ALLOWAPLE NUMBER OF ITERATIONS
                                                                             ZAN10080
                  TTMAX
C
                              PER ROOT (INPUT)
                                                                             Z4N10090
C
C
                          - AN INTEGER VECTOR OF LENGTH .GE. KN+N.
                                                                             ZAN10100
                  INFER
                              PUTPUT INFER(J) CONTAINS THE NUMBER OF
                                                                             ZAN10110
                              THERATIONS USED IN FINDING THE J-TH ROOT
C,
                                                                            ZAN10120
C
                              WHEN CONVERGENCE WAS ACHIEVED. IF
                                                                             ZAN10130
C
                              CONVERGENCE WAS NOT CHTAINED IN ITMAX
                                                                             ZAN10140
C
                              ITERATIONS, INFER(J) WILL CONTAIN ITMAX+1
                                                                             ZANIC150
C
                              (MUTPUT)
                                                                             ZAN10160
ſ,
                  IER
                         - FREDR PARAMETER (OUTPUT)
                                                                             Z4N10170
٢
                           WARNING FRROR = 32 + N
                                                                             ZAN10180
                              N = 1 FAILURE TO CONVERGE WITHIN ITMAX
•
                                                                             ZAN10190
C
                              ITERATIONS FOR ONE OF THE (N) NEW ROOTS TO
                                                                            ZAN10200
                                                                            ZAN10210
                              BE FOUND
C
                         - DULIBLE
C
    PRECISION
                                                                             Z4N10220
    RED'D IMSL ROUTINES - UFPIST
                                                                            Z4N10230
    AUTHOR/IMPLEMENTOR
                         - n. G. JOHNSCHILL L. WILLIAMS
                                                                            ZAN10240
C
    LANGUAGE
                         - FORTRAN
                                                                            Z4N10250
                                                                            . ZAN1 0260
                         - SEPTEMBER 1, 1971
    LATEST REVISION
                                                                            Z#N10270
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C
                                                                                 ZAN10280
       SUBPOUTTNE ZANLYT
                             (F.FPS.NSIG.KN, NGUESS, N, X, ITMAX, INFER, IER)
                                                                                 ZAN10290
       COMPLEX#16
                             X(1), CNE, D, DD, DEN, DI, FPRT, FRT,
                                                                                 ZAN10300
      1
                             H,RT,T1,T2,T3,TEM,X0,X1,X2,BI,F,XX
                                                                                 ZAN10310
       DOUBLE PRECISION
                             OZ, FPS, FPS1
                                                                                 ZAN10320
       DIMENSION
                             INFER(1)
                                                                                 ZAN10330
       IEP = 0
                                                                                 ZANIC340
       3VE = (1.00+00, 0.00+00)
                                                                                 ZAN1 03 50
       EPS1 = 10.0D+00**(-NSIG)
                                                                                 ZAN10360
       TODNJ = 0
                                                                                 ZAN10370
       IROMB = 0
                                                                                 ZAN10380
C
                                       SET NUMBER OF ITERATIONS
                                                                                 ZAN10390
       MB1 = KN+1
                                                                                 ZAN10400
       MR2 = KN+N
                                                                                 ZAN10410
       LSTART = MB2+1
                                                                                 ZAN10420
       MPG = MBI+NGUESS
                                                                                 ZAN10430
       DD 2 I = MPG, MB2
                                                                                 ZAN10440
          X(I) = (0.0+0,0.0+0)
                                                                                 ZAN10450
       L = MB1
                                                                                ZAN10460
       IF (KN .EQ. 0) GO TO 5
                                                                                ZAN10470
       DO 3 I = 1, KN
                                                                                ZAN10480
          INFER(I) = 0
                                                                                Z 4 N1 0 4 90
          TTEMP = MB2+I
                                                                                Z 4 N1 0500
          X(ITEMP) = X(I)
                                                                                Z4N10510
          ITEMP = MR2+ITEMP
                                                                                ZAN10520
     3 \times (ITEMP) = \times (I)
                                                                                ZAN1 0530
     5 JK = 0
                                                                                Z 4410540
       QZ = CDABS(X(L))
                                                                                ZAN10550
       IF (QZ .LE. 1.0D-15) GO TO 25
                                                                                ZAN1 0560
C
                                       ROCT ESTIMATE NOT EQUAL TO ZERO
                                                                                ZAN10570
   10 RT = (.90+00,0.00+00)*X(L)
                                                                                ZAN10580
       ASSIGN 15 TO NN
                                                                                ZAN10590
       G7 T7 135
                                                                                ZAN10600
   15 XO = FPRT
                                                                                ZAN10610
       RT = (1.10+00+0.00+00)*x(L)
                                                                                Z4N10620
       ASSIGN 20 TO NN
                                                                                ZAN1.0630
       GO TO 135
                                                                                ZAN1 0640
   20 X1 = FPRT
                                                                                ZAV10650
       H = X(L)-RT
                                                                                ZAN10660
       RT = X(L)
                                                                                ZAN10670
       ASSIGN 40 TO NN
                                                                                ZAN10680
       G7 T7 135
                                                                                Z 4 N 1 0 5 9 0
C
                                       ROOT ESTIMATE EQUAL TO ZERO
                                                                                ZAN19700
   25 RT = -ONE
                                                                                ZAV10710
       ASSIGN 30 TO NN
                                                                                ZAN10720
       GC TO 135
                                                                                ZAN19730
   30 XO = FPRT
                                                                                ZAN10740
       PT = ONE
                                                                                Z4N10750
       ASSTON 35 TO NN
                                                                                Z4N10760
      G7 T7 135
                                                                                Z 4N1 0770
   35 X1 = FPRT
                                                                                ZAN10780
      P^{T} = (0.00+00,0.00+00)
                                                                                ZAN10790
      H = -DNE
                                                                                CORPINAZ
      ASSIGN 40 TO NN
                                                                                Z4N10810
      37 TO 135
                                                                                Z4V10920
   40 X2 = FPPT
                                                                                Z 4N10830
   45 D = (-0.50+00.0.00+00)
                                                                                ZAN1 0840
C
                                       BEGIN MAIN ALGORITHM
                                                                                Z4410850
   50 DD = PNE + D
                                                                                C6801447
      T1 = \chi 0 * 0 * 0
                                                                                Z4N10870
      T2 = X1+00+00
                                                                                Z & NI DA 80
                                         66
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```
XX = X2 + DD
                                                                                                                                                                   ZAN1 0890
             T3 = X2*D
                                                                                                                                                                   Z 441 0900
             RI = T1-T2+XX+T3
                                                                                                                                                                   ZAN10910
             DEN = RI + RI - (4.00 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0 
                                                                                                                                                                   Z4N10920
                                                                               USE DENOMINATOR OF MAXIMUM AMPLITUDE ZAVIDGED
             T1 = CDSQRT(DEN)
                                                                                                                                                                    C4P0 INAS
             T2 = RT + T1
                                                                                                                                                                   Z 4111 0950
             T3 = RT - T1
                                                                                                                                                                   ZAN10960
             OZ = CDARS(T2) - CDARS(T3)
                                                                                                                                                                   ZAN1 0970
             IF (02 .GE. 1) GO TO 60
                                                                                                                                                                   Z AN10980
       55 DEN = T3
                                                                                                                                                                   ZAN10990
             GC TO 65
                                                                                                                                                                   ZANIIOCC
      60 DEN = T2
                                                                                                                                                                   Z4N11017
C
                                                                               TEST FOR ZEED DENOMINATOR
                                                                                                                                                                   ZAN11020
       65 QZ = CDARS(DEN)
                                                                                                                                                                   ZAN11030
             TF (02 .GT. 1.0-15) GO TO 75
                                                                                                                                                                   ZAN11040
      70 DEN = ONF
                                                                                                                                                                   ZAN11950
       75 DT = ((-2.00+00.00+00)*XX]/DEN
                                                                                                                                                                   Z3V11060
             H = DI + H
                                                                                                                                                                   ZAN11070
             RT = RT + H
                                                                                                                                                                   Z4N11080
                                                                               CHECK CONVERGENCE OF THE FIRST KIND
                                                                                                                                                                  ZAV11090
             QZ = CDABS(H/PT)
                                                                                                                                                                   ZAN11100
             TF (02 .LE. EPS1) 60 TO 100
                                                                                                                                                                   Z4N11110
      80 ASSTON 85 TO NA
                                                                                                                                                                   ZANII120
             go To 135
                                                                                                                                                                   ZAN11130
      85 OZ = CDABS(FPRT)-CDABS(X2*(10.0DC.0.CDC))
                                                                                                                                                                   Z4N11140
             TF (DZ .LT. 0.D+0) 60 TO 95
                                                                                                                                                                   ZAN11150
                                                                               TAKE REMEDIAL ACTION TO INDUCE
                                                                                                                                                                   ZAN11160
                                                                                   CONVERGENCE
                                                                                                                                                                  ZAN11170
      90 DT = DI*(0.5D+00,0.00+00)
                                                                                                                                                                   ZAN11180
             H = H*(0.50+00,0.00+00)
                                                                                                                                                                   ZAN11190
             RT = RT-H
                                                                                                                                                                  ZAN11200
             G2 T3 135
                                                                                                                                                                  Z4N11210
      95 \times 9 = 11
                                                                                                                                                                  Z4N1122C
             X1 = X2
                                                                                                                                                                   ZAN11230
             X? = FPRT
                                                                                                                                                                   ZAN11240
             D = DI
                                                                                                                                                                  ZAN11250
             30 TO 50
                                                                                                                                                                  ZAV11260
                                                                             A PORT HAS REEN FOUND
                                                                                                                                                                  ZAN11270
   100 FRT = F(RT)
                                                                                                                                                                  ZAN11280
    105 X(L) = RT
                                                                                                                                                                  ZAN11290
             TTEMP = MRZ+L-TRCMR
                                                                                                                                                                  ZAN11300
             X(ITEMP) = PT
                                                                                                                                                                  ZAN11310
             TTEMP = M82+M82+L
                                                                                                                                                                  Z4411320
             X(TTEMP) = RT
                                                                                                                                                                  ZAN11330
                                                                              CHECK TO SEE TE COMPLEX-CONJUGATE
                                                                                                                                                                  ZAN11340
                                                                              TS ALSO A ROOT
                                                                                                                                                                  ZAN11350
             IF (CDARS(F(DCCNJG(X(L)))) .GT. 10.D+0*CDARS(FRT)) GO TO 115
                                                                                                                                                                  ZAN11360
             QZ = SDARS(X(L) - DCDMJG(X(L)))
                                                                                                                                                                  ZAV11370
             TE (TOON) .NE. 0 .PP. OZ .LT. ).OD-8) GO TO 115
                                                                                                                                                                  ZAN11380
             TSTAPT = L+2
                                                                                                                                                                  ZAN11390
             THISER! = L+1
                                                                                                                                                                  ZAN11400
                   D7 110 INSERT = TSTART, MR 2
                                                                                                                                                                  ZAM11410
                   X(INSERT) = X(INSEP1)
                                                                                                                                                                  ZAN11420
                   TUCFRI = INSERT
                                                                                                                                                                  Z 64111430
            X(L+1) = D^{r} \cap NJG(X(L))
                                                                                                                                                                  ZAN11445
             1 = 1 INC 1
                                                                                                                                                                  ZAN11450
            $7 TT 120
                                                                                                                                                                  Z4N11460
   115 TON'J = 0
                                                                                                                                                                  Z8N11470
   120 CONTINUE
                                                                                                                                                                  ZAN11480
   125 [VFF?(L) = JK
                                                                                                                                                                  ZAN11490
```

```
24411500
      L = L+1
      TF (L .LF. MB2) GO TO 5
                                                                            ZAN11510
                                     PETURY TO CALLING PROGRAM
                                                                            ZAN11520
  130 30 TO 185
                                                                            ZAN11530
  135 JK = JK+1
                                                                            ZAM11540
  TF (JK .GT. (TMAX) GD TO 180
140 FRT = F(RT)
                                                                            Z4111550
                                                                            ZAN11560
      FPRT = FRT
                                                                            Z 4N11570
                                     TEST TO SEE IF FIRST COUT IS HEING
                                                                            ZAN11580
                                        DETERMINED
                                                                            ZAV11590
      IF (L .FO. 1) 60 TO 160
                                                                            ZAN11500
      IF (L .LE. IROMB+1) GO TO 160
                                                                            ZAN11610
•
                                    COMPUTE DENOMINATOR FOR MODIFIED
                                                                            ZAN11620
C
                                        FUNCTION
                                                                            ZAM11630
  145 LTMUP = MR2+L-TRCM9-1
                                                                            74411640
      D7 150 I = LSTART, LIMUP
                                                                            ZAN11650
         TEM = RT - X(T)
                                                                            ZAN11660
         QZ = CDARS(TEM)
                                                                            Z4N11670
         TF (0Z .LT. 5.00-15) GO TO 175
                                                                            ZAN11580
  150 FPRT = FPRT/TEM
                                                                            ZAM11590
                                    THETK CONVERGENCE OF THE SECOND KIND ZUMIITOD
  160 QZ = CDABS(FRT)
                                                                            Z4N11710
      IF (OZ .GE. EPS) GO TO 170
                                                                            Z 4N11720
  165 QZ = CDARS(FPPT)
                                                                            24411730
      TF (02 .LT. EPS) 00 TO 105
                                                                            Z4N11740
  170 GO TO NN. (15,20,30,35,40,85)
                                                                            ZAV11750
  175 RT = RT + (1.0000010+0,0.60+0)
                                                                            ZAN11760
      G7 TF 135
                                                                            ZAN11770
                                    WARNING FEDOR . ITMAX = MAXIMUM
                                                                            Z 1N11780
 180 TF9 = 33
                                                                            ZAN11790
      INFER(L) = ITMAX + 1
                                                                            ZAN11800
      TROMR = IROMR + 1
                                                                            Z4V11810
      X(L) = (12345678.123456780+0.12345678.123456780+0)
                                                                            ZAM11320
      TTEMP = MR2 + MR2 + L
                                                                            ZAN11830
      X([TEMP] = RT
                                                                            Z ^ 11 1 8 40
      L = L+1
                                                                            Z4N11850
      IF (L .LE. MR2) GO TO 5
                                                                            ZAN11860
 185 (F (IFR .ED. 0) 50 TO 9005
                                                                            Z4N11870
9000 CHATTAUF
                                                                            78811M65
      CALL UERTST (TER. ZANLYTI)
                                                                            ZAVITAGO
9005 RETURN
                                                                            Z#N11900
      FND
                                                                            ZAM11910
```

```
.....UEP11220
                                                                          UEP11931
    FUNCTION
                        - FORDE MESSAGE GENERATION
                                                                          UFF 11940
                        - CALL HERTST (JER, INAMEXXI)
                                                                          UEP11950
    USAGE
                         - EPERE PARAMETER. TYPE + N. WHERE
                                                                          UFC 11960
    PARSMETERS
                 IER
                             TYPE= 128 THOLTES TERMINAL FRAME
                                                                          JER11970
                                    64 IMPLIES WAFHING WITH FIX
                                                                          UFR11980
                                    32 IMPLIES WARNING
                                                                          UFP 11490
                                 = FRECE CODE FELFVANT TO CALLING FOUTINEJERIZORO
                  NAMEXX - NAME OF THE CALLING POUTINE
                                                                          JE912310
    AUTHOR/TMPLEMENTER
                        - PEDER SVENDSEN
                                                                          JER 12020
                         - FORTRAN
                                                                          JEP12030
    LANGUAGE
                                                                         .UEF 12040
                      - JANHARY 19, 1971
    LATEST REVISION
                                                                          11=0 1 20 50
                                                                          UEF12060
      SURPRUTINE UFRIST (TEP . NAME)
                                                                          UF9 12070
                                                                          JFR 12080
                          ITYP(5,4), [R]T(4)
      DIMENSION
                                                                          UEP12090
                          NAME (3)
                                                                          UFR 12100
      INTEGER#2
      THIEGER
                          WERN, WEDF, TERM, PRINTE
                                                                          JEP12110
      FOUTVALENCE
                          (!PIT(1), WARN), (IBIT(2), WARE), (IRIT(3), TERM)
                                                                          UF212120
                                                            ٠,
               TTYP
                          ALMADMIT TANG 1 . 1 . 1 . 1 . 1
                                                                          UEF 121 30
                                                            ٠,
                          *WARN*, *ING(*, *WITH*, * FIX*, *)
                                                                          UEC 12140
                          TERMI, THALI,
                                                     1,1
                                                                          UER12150
                          'NON-', 'DEFI', 'NED ','
                                                                         JER 1 21 60
               TRIT
                          / 32,64,128,0/
                                                                         UE212170
      DATA
               PRINTP
                                                                         UFP 1 21 80
      TED 2= TED
                                                                         JER12190
      IF (IF92 .GE. WARN) GO TO 5
                                                                         NEE 15500
                                                                         UFR 1.2210
ņ
                                   NON-DEFINED
      TER1=4
                                                                         UER12220
      GO TO 20
                                                                         UEP12230
   5 IF (!FR2 .LT. TEPM) GO TO 10
                                                                         UEF 1 2240
                                   TEPMINAL
C
                                                                         UFF12250
      TEP1=3
                                                                         UF712260
      30 TO 20
                                                                         UFR 1 2270
     TE (IER2 .LT. WARE) GO TO 15
                                                                         HFP12290
 10
                                   WARNING (WITH FIX)
                                                                         UE012290
                                                                         1EP12300
      [F? [=?
      G2 T7 20
                                                                         UEP12310
                                   WARNING
                                                                          PED 15350
    TER1=1
  15
                                                                          JER1 2333
                                   FXTPACT INI
                                                                         UFS12340
  20 TER 2=TER 2-I RIT (TER1)
                                                                         UF9 1 2350
                                   DETHT ERROR MESSIGE
                                                                         JF912360
      WRITE (PRINTR, 25) (ITYP ('T. TEP1), I=1,5), NAME, IER 2
                                                                         UE912370
  25 FORMAT( *** I M S L(UFRTST) *** 1,504,4X,367,4X,12)
                                                                         UER12380
      RETURN
                                                                         UF012390
      END
                                                                         UER 12400
```



	SUPPOUTINE RSLUZ(X, FJ, NMAX, A, ND. TEPP, FJ&PRX, RF)	20012410
	IMPLICIT REAL +8 (A-H, O-Z)	00012420
	DIMENSION FU(1), FURPRX(1), FR(1)	0112430
	NAXY=VAX	77012440
	TE(NMAXT.GE.1)GD TO 31	00012450
	TF(DABS(A).LF.1.00-15)00 TO 10	20012460
	GD TD 20	00012470
10	TERR=4	00012480
	RFTLIRV	00012490
50	NMAX BR= I ABS (NMAXT)	00012500
	VMA XT=1	00012510
30	\$F(A.GT.0.0)GO TO 40	00012520
	[F(D&BS(A).LF.1.20-15)GC TC 42	00012530
	[FRP=1	00012540
	RETURN	00012550
40	TF(A.LT.1.3)GO TO 70	00012560
	[FRP=2	00012570
	RETURN	00012580
70	IF(X.GT.0.7)GO TO 130	00012590
	IERR=3	00012500
	RETURN	00012610
130	1 = 0 = 0	00012620
	FPS( N= .500*10. **(-ND)	99912639
	NMP1 = NMAX+1	00012640
	07 160 N=1,NMP1	00012650
160	FJAPRX(N)=0.0	00012669
	SJM=(X/2.)**A/DGAMMA(1.+A)	20212670
	D1=2.3026D0*N2+1.3863D0	00012680
	IF(NYAXT.LF.0)GC TC 230 Y=.50C*O1/NMAXT	00012690 00012700
	CALL TZ(Y, TANS)	00012710
	P=VMAXT*TAVS	00012710
	GO TO 240	00012730
230	3=0.0	70012740
240	Y=.73576D9*D1/X	00012750
	TALL TZ(Y, TANS)	00012760
	S=1.359100+X+TANS	00012770
	1=(R.GT.S)GN TO 280	00012780
	NII=1+TDTNT(S)	00012790
	קר דר באַ 190	00012800
280	NU=1+[D[NT(0)	20012810
290	M = 0	00012820
	FL=1.	00012930
	LIMIT = (NU/2)	00012840
320	4=4+1	00012850
	FL=FL*(M+4)/(M+1.00)	00012860
	IF(M.LT.LIMIT)GO TO 320	20012870
	N=2+M	30012880
	p=0.0	00012890
	S=0.0	00012900
390	DENUM=5.*(#+N)/X-B	20012912
	IF(DARS(DENOM).LE.1.0D-15)DENOM=DENOM+1.0D-15	20012920
430	Ral./DENOM	00012930
	VM^∂2=MOD(N, 2)	20012940
	IF(NMOD2.NE.O)GO TO 480	22012950
	FL=FL+(N+2.D0)/(N+2.*4)	00012960
	FLMRDA=FL*(N+^)	00012970
	50 TO 490	00012980 00012990
480	FLMRD&=0.0	00012990
490	S=R#(FLMBD8+5) [F(N.LE.NMAXT)FR(N)=P	00013010
	70	

	N= V-1	00013020
	TE(N.GE.11GD TD 391	20013232
	FJ(1)=SUM/(1.+5)	00013040
	IF(NMAXT.EG.O)CO TO 570	00013050
	00 560 N=1.NMAXT	22013060
560	FJ(N+1)=R?(N) *FJ(N)	20213270
570	07 640 M=1.NMP1	00013080
	IF(DARS((FJ(N)-FJAPRX(N))/FJ(N)).LE.FPSLON)GO TO 640	00013090
	97 610 M=1,NMP)	2013100
610	FJ&PRX(M)=FJ(M)	00013110
	NIJ=NIJ+5	20013120
	GO TO 290	20013132
640	CONTINUE	22213142
	IF(NMAX.GE.O) PETIJRN	20013150
	FJ(?)=2.+A+FJ(1)/X-FJ(2)	00013160
	TE(NMAXAB.EO.1)RETURN	20213172
	D2 650 N=2.NMAXAP	00013180
650	FJ(N+1)=2.*(A-N)*FJ(M)/X-FJ(N-1)	20013190
	RETURN	20013200
	END	00013210

```
SURPRUTINE BSCJZ(X,Y,U,V,MMAX,A,ND,1ERF,UAPPEX,VAPPEX,RF1,RP2)
                                                                               BSC13220
      IMPLICIT REAL +8 (A-H. 7-Z)
                                                                               35013230
      DIMENSION U(100).V(100).UAPPPX(100).VAPPPX(100).RRI(100).
                                                                               95013240
                 RP2(100)
     1
                                                                               +SC13250
      IF(#.GE.O.O)GC TO 40
                                                                               BSC13260
      [FOC=]
                                                                               351 13270
      PETURY
                                                                               BSC13290
                                                                               35013290
40
      TFIA.LT.11GO TO 70
      TEOD=?
                                                                               ASC13300
      RETURN
                                                                               9SC13310
      TE(X.GT.O. 1)GD TO 110
                                                                               35013320
70
      TE( D&BS(Y) . LE . 1 . 00-14) GP TO 90
                                                                               BSC13330
      G7 T7 110
                                                                               35C13340
                                                                               35013350
BSC13360
90
      TERP=3
      DETURN
                                                                               85013370
      TE(NMAX.GE.O)GO TO 140
110
      IFPR=4
                                                                               35013380
                                                                               85013390
      RETITAN
      TFPR=0
                                                                               85013400
140
      =PSL ON=. 500+1 0.++(-ND)
                                                                               85013410
      NMP ! = YMAX+!
                                                                               BSC13420
                                                                              35713430
95013440
      00 200 N=1+NMP1
      HAPPOX(N)=0.0
200
      C.C.(N)XPAAAV
                                                                               95013450
      YI = DABS(Y)
                                                                              357 13460
      722=X**2+Y**?
                                                                               85013470
      RZ=DS DQT ( 9Z2 )
                                                                              85013480
      TF(DARS(X).LE.1.09-14)60 TO 290
                                                                              35013490
      PHT=DATAN2(Y1.X)
                                                                               35013500
      IF(X.LT.0.3) PHI=3.14159265358979300 + PHI
                                                                              95013510
      37 77 300
                                                                              35013520
290
      PHI=1.570796326794896D0
                                                                              BSC13531
      C=DEXP(Y1)+(RZ/2.)++A/DGAMMA(1.+A)
370
                                                                              35013540
      SIM2=# *PHT-X
                                                                              35013550
      SUM1 = C + DCOS (SUM2)
                                                                              BSC13561
      SUM2=C*DSTN(SUM2)
                                                                              35013570
      D1=2.302600*ND+1.286300
                                                                              PSC13580
      IF(NMAX.GT.O)GO TO 380
                                                                              85013590
      o = 0 • 0
                                                                              BSC13600
      GD TO 390
                                                                              85C13610
                                                                              35°1362°
      PARAM=.500+D1/NMAX
380
      SALL TZ(DARAM, TANS)
                                                                              85013630
      R=VMAX+TAVS
                                                                              95013640
390
      S=1.359100*9Z
                                                                              35013650
      PARAM= . 735760 0+(01-Y1) /RZ
                                                                              BSC12660
      CALL TZ (PARAM, TANS)
                                                                              95013670
      TF(Y) .LT.D1) S=S+TANS
                                                                              35713680
      TF(P.GT.S)60 TO 450
                                                                              BSC1 3690
      N'J=1+[DINT(S)
                                                                              946131CO
      30 TO 460
                                                                              95C13710
450
      VJ=1+IDINT(R)
                                                                              85C13720
                                                                              35-1373-
460
      N = 0
      FL=1.
                                                                              85013740
      rt=1.
                                                                              BSC13752
      C2=0.
                                                                              35013760
500
      V=V+1
                                                                              95013770
      FL=FL+(N+2.*8)/(Y+1.D0)
                                                                              35013780
      C=-Cl
                                                                              35013790
      21=12
                                                                              95013800
                                                                              45013810
      C2=C
                                           72
      TE(N.LT.NU)GO TO 500
                                                                              ASC13820
```

```
95013930
      R1=0.0
                                                                              35C13840
      R2=0.0
      51=0.0
                                                                              BSC13850
      52=0.0
                                                                              BSC13860
      C=(2.*(A+N)-X*R1+Y1*P2)**2+(X*P2+Y1*R1)**2
                                                                              BSC13870
      21=(2.*(A+V)*X-5Z2*P1)/C
                                                                              95C13880
      R2=(2.*(A+N)+Y1+9Z7+92)/C
                                                                              ASC13890
                                                                              BSC13900
      FL=FL+(N+1.D^)/(N+2.+4)
      C=2. +(N+A) +FL
                                                                              3SC13910
                                                                              95013920
      FLAMB1=C+C1
      FLAMR2=C+C2
                                                                              BSC13930
                                                                              BSC13940
      0=01
      C1=-C2
                                                                              35C13950
                                                                              BSC13960
      C2=C
      S=91+(FLAMB1+S1)-P2+(FLAMB2+S2)
                                                                              BSC13970
      $2=01*(FLAMR2+$2)+P2*(FLAMR1+$1)
                                                                              35013980
                                                                              3SC13990
      51=5
      IFEN.GT.NMAXIGO TO 770
                                                                              BSC14000
                                                                              95014010
      PR1(N) =R1
                                                                              95C14020
      22(V)=R2
                                                                              BSC14030
770
      1-121
      TE(N.GE.1)G0 TO 610
                                                                              9 SC 14040
      S=(1.+S1)**2+S2**2
                                                                              BSC14250
                                                                              95014062
      U(1)=(SUM1+(1.+S1)+SUM2+S2)/C
      V(1)=(SUM2+(1.+S1)-SUM1+52)/C
                                                                              BSC14070
                                                                              BSC14080
      IF(NMAX.50.0)GO TO 850
      07 840 N=1 NMAX
                                                                              35014090
      IJ(N+1) = RR1(N) + U(N) - RP2(N) + V(N)
                                                                              BSC14100
      V(V+1)=PR1(V)*V(N)+PPC2(M)*U(M)
                                                                              8551+110
840
      JE(Y.LT.0.1)GC TO 860
                                                                              9 50 141 20
850
      088 CT CP
                                                                              45C14130
      D? 870 N=1, NMP1
                                                                              85C14140
860
      V(N) = -V(N)
                                                                              85014150
870
                                                                              85014160
880
      D7 950 N=1, NMP1
      TEMP1=(IJ(N)-IJ&PPRX(N)) ++2
                                                                              B SC 14170
      TEMPL=TEMP1+(V(N)-VAPPRX(N))++2
                                                                              35C14180
                                                                              BSC14190
      TEMP1=TEMP1/(U(N) **2+V(N) **2)
                                                                              35014200
      (F(TEMP).LE.EPSLON)GO TO 950
                                                                              BSC14210
      DO 920 M=1, NMP1
      UAPPRX (M)=U(M)
                                                                              95014220
      VAPPRX(M)=V(M)
                                                                              BSC14230
920
                                                                              BSC14240
      VII=NU+5
                                                                              BSC14250
      G2 T3 460
      CONTINUE
                                                                              35014260
950
      RETIRV
                                                                              BSC14270
                                                                              BSC14280
      SUBPRUTIME TZ (Y,TAPS)
                                                                              35014290
      REAL *8 Y, Z, P, TANS, DLOG
                                                                              3SC14300
      #F(Y.GT.10.0)60 TO 40
                                                                              BSC14510
      P=. 00005794100* Y-.0017614800
                                                                              85114320
                                                                              BSC14333
      P=Y*P+.0208645D0
                                            Reproduced from best available copy.
      P=Y*P-.12901300
                                                                              35014349
                                                                              95C14350
      P=Y*7+.8577700
                                                                              35014360
      TANS=Y*P+1.1012500
                                                                              95214370
      RETURN
40
      Z=DL3G(Y)-.77500
                                                                              45C14380
                                                                              05/14390
      P=(.77500-DLOG(Z))/(1.0+Z)
                                                                              91714400
      TANS=Y/((1.+P)+Z)
      PFTIJON
                                                                              95014410
                                           73
      FND
                                                                              35514420
```

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